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ABSTRACT

This is the teacher's manual to accompany part 1 of the workbook for grade 7 pupils in the Comprehensive Instructional Management System, a teacher-designed unified mathematics curriculum for kindergarten through grade 7. It is composed of a curriculum, a testing component, and a computer-managed reporting system. The curriculum is organized into modules of similar skills or understandings from 14 topics. Each module has: a suggested period of duration for student mastery; a listing of new skills to be mastered by the student in the form of performance objectives; a section devoted to vocabulary, terms, and symbols; getting-started activities on prerequisite skills in preparation for undertaking new objectives; strategies for teaching the new skills; and student activities for providing reinforcement for the newly learned objective. After the sequence of modules and content by topic are displayed, teaching suggestions for 58 modules on 49 performance objectives are given. The content is from the topics of: numbers and numeration; addition; subtraction; fractions; geometry; measurement; algebraic concepts; graphs, probability, and statistics; and problem solving, plus review. Pages from the student's workbook are interspersed with the suggested activities. (MNS)

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CIMS

COMPREHENSIVE INSTRUCTIONAL MANAGEMENT SYSTEM

A Cyclical Mathematics Curriculum

PART 1—TEACHER

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FOREWORD

The Comprehensive Instructional Management System (CIMS) is a unified mathematics curriculum for grades K-7. The curriculum is cyclical in design, and reflects the most recent New York City and New York State guidelines.

Utilizing the experience of our system's teachers, we have developed a teacher-designed curriculum that includes a rich selection of classroom-tested successful mathematics activities. The major goal of the CIMS mathematics project is to provide the classroom teacher with a well-structured, objective-based curriculum. All objectives are supported by a variety of strategies. Student mastery of these objectives is readily diagnosed through the system's testing component.

The curriculum format is structured to allow the teacher to determine the ability level of the students in the topic under study, and to permit the students to move forward at their individual rates. Each module of instruction provides the teacher with a set of related performance objectives. These objectives are supported by activities which probe and develop the students' readiness to undertake new skills and concepts. Further reinforcement is given to mastery of the objectives through accompanying student workbooks.

It is hoped that this curriculum will not only assist teachers in developing vital mathematics instruction, but will serve to expand and enrich students' mathematics skills.

Charlotte Frank

Charlotte Frank,
Executive Director
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ACKNOWLEDGMENTS

The Comprehensive Instructional Management System (CIMS) was developed as a project of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director. Dr. Peter Negroni, Community School Superintendent CSD 12; Dr. Harvey Garner, Community School Superintendent CSD 18; Carlos Edwards, Community School Superintendent CSD 23, and Joseph Pacheco, Community School Superintendent CSD 32 supervised the overall development of the CIMS program within their respective districts.

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INTRODUCTION

The Comprehensive Instructional Management System (CIMS) is a teacher-designed educational delivery system composed of a curriculum, a testing component and a computer managed reporting system.

Designed to support mathematics instruction in the classroom, CIMS provides a comprehensive cyclical mathematics curriculum for grades K through 7. Each manual describes the curriculum for a single level or grade. Teacher curriculum developers from elementary, intermediate and senior high schools utilized the most current New York City and New York State guidelines in formulating the curriculum's modules and sequence.

Curriculum Topics

The CIMS mathematics curriculum has been organized into the following 14 topics:

.01 Numbers and Numeration
.02 Addition of Whole Numbers
.03 Subtraction of Whole Numbers
.04 Multiplication of Whole Numbers
.05 Division of Whole numbers
.06 Fractions
.07 Decimals
.08 Integers
.09 Geometry
.10 Measurement
.11 Algebraic Concepts
.12 Graphs, Probability, Statistics
.13 Problem Solving
.14 Review

Modules of Instruction

The CIMS curriculum is organized into modules or groupings of similar skills or understandings. Each module has a suggested period of duration for student mastery, a listing of new skills to be mastered by the student in the form of performance objectives, a section devoted to vocabulary, terms and symbols, getting started activities which serve as prerequisite skills in preparation for undertaking new objectives, strategies for teaching the new skills and student activities for providing reinforcement for the newly learned objective. Note that the lesson duration is included only as a guideline to the teacher for planning purposes. The actual time required for each module will vary from class to class and from child to child depending on the needs of the students and the teacher's judgment.

Priorities of Performance Objectives

There is an order of priority to be recognized in teaching the content presented in the CIMS NEW YORK CITY MATHEMATICS CURRICULUM. Performance objectives with the greatest priority are so basic that unless the student acquires a satisfactory degree of competency in them, the student cannot progress. Such topics are marked A and mastery by all pupils is essential.

Performance objectives marked B are required to be presented to all students. Mastery is not expected at this time. These objectives will be developed further and reinforced later.

Performance objectives marked C are of special interest. These should be taught to increase positive attitudes towards mathematics.

Summary: A - Mastery of this objective is essential for progress. Opportunities for reinforcement should be provided throughout the school year.

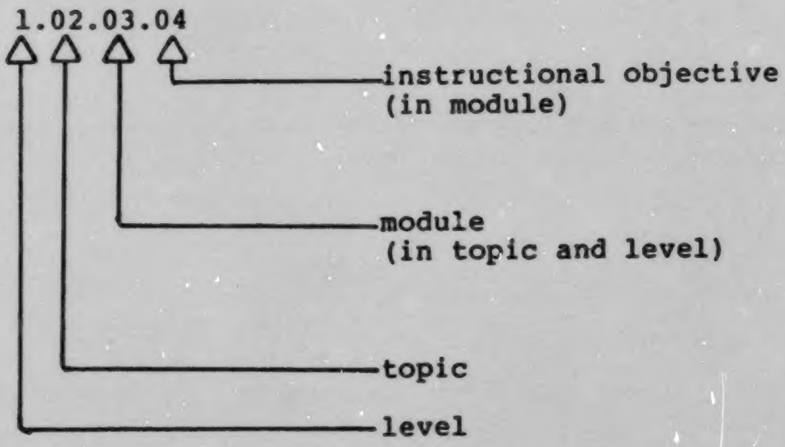
B - These objectives are to be taught to all pupils. Mastery is not essential at this time because the topics will be further developed and reinforced.

C - Objectives of special interest.

Topic Tab Key

01. Numbers and Numeration
02. Addition of Whole Numbers
03. Subtraction of Whole Numbers
04. Multiplication of Whole Numbers
05. Division of Whole Numbers
06. Fractions
07. Decimals
08. Integers
09. Geometry
10. Measurement
11. Algebraic Concepts
12. Graphs, Probability, Statistics
13. Problem Solving
14. Review

Module Coding Key


1.02.03.04
instructional objective (in module)
module (in topic and level)
topic
level

INSTRUCTIONAL SEQUENCE DESCRIPTION

An instructional sequence is given in list form on the following pages for this level. The sequence has been arranged to reflect what is considered the most effective presentation of the mathematics curriculum. Each module in the list is described by its unit number, module number, title, and page location.

The CIMS tests are integrated within the manual sequence. The test number is located in the first column. Each test is designed to assess priority objectives from the preceding modules. The tests are used to determine student mastery of specific performance objectives within these modules and appear in a multiple choice format. Each performance objective code number preceded by an * indicates that the performance objective is not tested on a CIMS test.

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INSTRUCTIONAL SEQUENCE - LEVEL 7

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
	1	7.11.01	Translating arithmetic word phrases into numerical phrases..................	1
	2	7.11.02	True and false numerical statements........	2
	3	7.14.01	Numbers through billions..................	4
7.01	4	7.02.01	Addition of whole numbers.................	5
	5	7.01.01	Palindromes...............................	6
	6	7.03.01	Subtraction of large numbers..............	7
	7	7.13.01	Solving a simpler but equivalent problem...	8
7.02	8	7.04.01	Multiplication of large numbers...........	9
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	10	7.01.02	Divisibility rules: 2, 5, 10..............	12
7.03	11	7.01.03	Divisibility rules: 3, 4, 6, 9............	13
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7.04	14	7.09.03	Triangles.................................	17
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INSTRUCTIONAL SEQUENCE - LEVEL 7

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	30	7.12.02	Probability with a spinner	41
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INSTRUCTIONAL SEQUENCE - LEVEL 7

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
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7.09	51	7.08.02	Multiplication of signed whole numbers.....	68
	52	7.08.03	Division of signed whole numbers...........	69
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Topic 03: **Subtraction of Whole numbers**

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Topic 04: **Multiplication of Whole Numbers**

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Topic 05: **Division of Whole Numbers**

7.05.01 Express a remainder as a fraction....... 11

Topic 06: **Fractions**

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7.06.02 Division with fractions................. 26
7.06.03 Ratio and proportion.................... 27
7.06.04 Meaning of percent...................... 46
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7.14.04 Permutations and combinations.............. 61
7.14.05 Positive and negative integers............. 66

Unit 1

7.11.01 TRANSLATING ARITHMETIC WORD PHRASES INTO NUMERICAL PHRASES

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.11.01.01A list the operations associated with words; e.g.,

sum	+
product	x
more than	>
decreased by	-
diminshed by	-

7.11.01.02A determine which operations have the commutative property; e.g.,

4 x 6 = 6 x 4	true
7 - 8 = 8 - 7	false

7.11.01.03A translate a word phrase expressing a binary operation into a mathematical expression. e.g.,

nine plus three = 9 + 3

the product of seven and thirteen = 7 x 13

7.11.01

VOCABULARY

TERMS	SYMBOLS
more than	>
less than	<
sum	+
product	x
difference	−
quotient	÷

GETTING STARTED

Write on the chalkboard.

e.g., $\begin{array}{r} 16 \\ \square\ \underline{25} \\ 41 \end{array}$ $\quad$ $\begin{array}{r} 83 \\ \square\ \underline{17} \\ 66 \end{array}$ $\quad$ $\begin{array}{r} 837 \\ \square\ \underline{11} \\ 9207 \end{array}$ $\quad$ $63 \ \square\ 9 = 7$

Ask a student to come to the chalkboard to fill in each operation symbol. Say, "How do you know what operation has been done?" Elicit that in subtraction or division, the answer is smaller than the numbers; in multiplication, the answer is much larger than the other numbers.

Repeat this procedure using other examples.

Say, "Name the four basic operations we use in math." Write them on the chalkboard, e.g., addition, subtraction, multiplication, division. Read a problem to the class.

e.g.,

> "Frank had 18 records. Three warped in the heat.
> How many are left?"

Ask a student which operation he/she would use to solve the problem.

Repeat this procedure using other verbal problems.

7.11.01

7.11.01.01 **list the operations associated with words**

STRATEGY 1: Write the following on the chalkboard.

Determine the arithmetic operation indicated in each phrase.

a) five plus seven
b) the product of eleven and two
c) the sum of three and eight
d) the quotient of twelve and four
e) seven decreased by three

Ask the students to write each of the above phrases using mathematical symbols, e.g., 5 + 7, 11 x 2, 3 + 8 ... 7 - 3. Ask the students what words are associated with addition. List them on the chalkboard. Continue this procedure for subtraction, multiplication, division, and relation symbols.

STRATEGY 2: Prepare a class set of cards. On each write one word or phrase associated with a mathematical operation, e.g. the sum of , diminished by the product of

Going around the classroom, display a card to each student and ask him/her to say the operation associated with the card, e.g., the sum of is associated with addition. Repeat this procedure until all of the students have had a chance.

7.11.01

7.11.01.02 **determine which operations have the commutative property**

STRATEGY 1: Write the following on the chalkboard.

Compute the sums in each pair of examples below.

	(A)	(B)
a)	12 + 19 = ☐	19 + 12 = ☐
b)	125 + 68 = ☐	68 + 125 = ☐
c)	57 + 35 = ☐	35 + 57 = ☐
d)	23 + 87 = ☐	87 + 23 = ☐

Elicit from the students that the examples in each pair are equal to each other, e.g., 12 + 19 = 19 + 12. Say to the students that the order of the addends does not affect the sum. Explain to the students that the property of addition, which says that the order in which we add two whole numbers does not affect the sum, is called the commutative property of addition.

Write the following on the chalkboard.

Compute the difference. 9 - 5, 5 - 9.

Say, "Are the differences the same?" Elicit that the differences are not the same, and that the order of the numbers does affect the result. Elicit that subtraction is not commutative.

Repeat this procedure for multiplication and division.

STRATEGY 2: Write the following on the chalkboard.

Add the two whole numbers as follows:

$$\begin{array}{r} 76 \\ \underline{+83} \end{array} \Big\downarrow \begin{array}{l} \text{add} \\ \text{down} \end{array} \qquad\qquad \begin{array}{r} 76 \\ \underline{+83} \end{array} \Big\uparrow \begin{array}{l} \text{add} \\ \text{up} \end{array}$$

Say, "Are the answers the same? This shows the commutative property of addition."

STRATEGY 2: Write the following on the chalkboard.

Compute the differences of the two whole numbers as follows:

$$\begin{array}{r} 69 \\ \underline{-25} \end{array} \downarrow \begin{array}{l} \text{subtract} \\ \text{down} \end{array} \qquad \begin{array}{r} 69 \\ \underline{-25} \end{array} \uparrow \begin{array}{l} \text{subtract} \\ \text{up} \end{array}$$

Say, "Are the differences the same?" Say that subtraction of two whole numbers is not commutative.

Repeat this procedure for multiplication and division.

7.11.01.03 **translate a word phrase expressing a binary operation into a mathematical expression**

STRATEGY 1: Write the following on the chalkboard.

Match the phrase in Column A with the correct phrase in Column B.

A	B
16 x 8	the difference between nine and five
16 - 8	sixteen diminished by eight
16 + 8	the product of sixteen and eight
$\frac{16}{8}$	the quotient of nine divided by five
9 - 5	sixteen divided by eight
9 + 5	nine times five
9 x 5	nine increased by five
9 ÷ 5	sixteen increased by eight

Say, "How can we translate each phrase?" Elicit that we use key words for symbols.

STRATEGY 2: Prepare a set of cards for each student with a verbal phrase on each, e.g., fifteen increased by 35. Divide the students into 2 teams, A and B. Ask one student from Team A and one from Team B to come to the chalkboard. Show one card. The student who writes the phrase as a correct mathematical expression first, gets a point for his/her team. Continue this procedure until each student has had a chance.

P.O.7.11.01.01

NAME: ______________________ CLASS: ________ DATE: ________

Put in the operational symbol associated with these words:

SUM

TOTAL

TWICE A NUMBER

QUOTIENT

PRODUCT

DIFFERENCE

DIMINISHED BY

SHARED BY

P.O. 7.11.01.02

NAME: ______________________ CLASS: ________ DATE: _______

I

Is the sentence 8 + □ = 2 + 8 true or false?

a) If □ =3 then 8 +□= 2 + 8 is ____________

b) If □ =7 then 8 +□= 2 + 8 is ____________

c) If □ =2 then 8 +□= 2 + 8 is ____________

d) If □ =9 then 8 +□= 2 + 8 is ____________

II 1. Write 2 multiplication facts with factors of 8 and 4. ______________________

2. Write two multiplication facts with a product of 7. ______________________

3. Write 4 multiplication facts with a product of 14.

____

III Match the commutative pairs

183 + 208

1,634 x 281

750 x 28

281 x 1634

28 x 750

208 + 183

P.O. 7.11.01.03

NAME: ______________________ CLASS: ________ DATE: _______

Translate each phrase into a numerical expression

1. The sum of seven and five 7+5

2. The difference between eight and five ________

3. The product of twenty and fifteen ________

4. The quotient of eighteen and nine ________

5. The sum of ten and six ________

6. Twelve decreased by two ________

7. Thirty-five increased by nine ________

8. Eight diminished by seven ________

9. Twenty plus eleven ________

10. The quotient of two and three ________

11. Four subtracted from eleven ________

12. Six more than three ________

7.11.02 TRUE AND FALSE NUMERICAL STATEMENTS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.11.02.01A state the order of operations as it applies to addition, subtraction, multiplication and division;
e.g.,

$$(\underbrace{6 \times 5}_{\text{①}}) + 2^2 + 30 \div 2 =$$

$$30 + \underbrace{2^2}_{\text{②}} + 30 \div 2$$

$$30 + 4 + \underbrace{30 \div 2}_{\text{③}}$$

$$\underbrace{30 + 4 + 15}_{\text{④}} = 49$$

*7.11.02.02A determine whether a numerical expression is a phrase or a sentence;
e.g.,

$4 \times 3 + 7$ is a phrase
$3 + 7 = 10$ is a sentence

7.11.02.03B translate a word sentence into a numerical sentence including parentheses;

e.g.,

Nine plus three is the same as six times two (9 + 3 = 6 x 2)

7.11.02.04A determine whether a numerical statement is true or false.

e.g.,

2 x 5 + 4 = 13 (false)
3 x 3 + 7 = 16 (true)
Add 2 to the product of 4 and 3 = (4 x 3) + 2 (true)

7.11.02

VOCABULARY

TERMS	SYMBOLS
order of operations	
numerical sentence	
numerical phrase	
parenthesis/ parentheses	( )
brackets	[ ]
braces	{ }

GETTING STARTED

Write the following problem on the chalkboard. Find the perimeter of a rectangle whose length is 12 feet and height is 7 feet. Say "What is the formula for perimeter?" Elicit $p = 2l + 2w$. Ask the students to use this formula to solve the problem.

e.g.,

$$\begin{aligned} p &= 2l + 2w \\ &= 2\times12 + 2\times7 \\ &= 24 + 14 \\ &= 38 \end{aligned}$$

The perimeter is 38 feet.

Repeat this procedure with other problems. Elicit the operations used to solve each problem, e.g., multiplication, addition.

Write the following problems on the chalkboard.

1. $6 + 8 = \square$
2. $7\times8 = \square$
3. $\frac{28}{7} + 4 = 8$
4. $8\times5 - 3 = 37$

Ask the students to solve examples 1 and 2 so that they are complete sentences. Ask two students explain the solutions to examples 3 and 4.

Repeat this procedure using other number problems.

7.11.02

7.11.02.01 **state the order of operations as it applies to addition, subtraction, multiplication and division**

STRATEGY 1: Write on the chalkboard.

e.g.,

Sara said, "Judy go to the store."
"Sara," said Judy, "go to the store."

Ask the students to read the sentences. Say "Do they have the same meaning? What is the difference?" Elicit that punctuation marks in English are used to make the meaning of a sentence clear. Write the mathematical sentence on the chalkboard, 6 + 7 x 2. Ask the students to compute the answer. Elicit at least two answers, e.g., 26 and 20. Ask two students to write their different solutions on the chalkboard.

e.g.,

6 + 7 x 2 = 13 x 2 or 6 + 7 x 2 = 6 + 14
= 26 = 20

Say that it is impossible to have two correct answers to the problem, therefore, mathematicians established a rule for the order of operations. Say that 20 is the correct answer because of the following rules which are called order of operations.The order of operations is as follows:

A.Multiplication and division in order from left to right;
B. Addition and subtraction from left to right.

Repeat this with other problems.

e.g.,

(1.) 7 x 3 + 4 = ☐
(2.) 8 - 4 ÷ 2 = ☐
(3.) 5 + 4 x 2 = ☐

STRATEGY 1: cont'd

Say that in the two beginning sentences we used punctuation marks. The use of punctuation helps to clarify a mathematical sentence.

e.g.,

$$(6 + 7) \times 2 = 13 \times 2 = 26 \quad \text{or} \quad 6 + (7 \times 2) = 6 + 14 = 20$$

Explain that in a mathematical sentence the operations within the parenthesis are done first before multiplying, dividing, adding or subtracting. Repeat this procedure with other problems.

e.g.,

$6 \times (4 + 3) =$ ☐

$3 + 4 \times (2 + 10) \div 2 =$ ☐

STRATEGY 2:

Say, "Let's pretend it is your best friend's birthday and you have invited him/her to go to the movies as your guest. Afterwards you are going to have ice cream, but each of you will pay for his/her own ice cream. You have saved $10.00 for this occasion. When you get to your friend's house, you discover that his/her little brother has to come also. Your friend has money for his brother's and his ice cream, but not the movies. If the ice cream costs $1.25 and the movies costs $2.50, will you have enough money?"

Write on the chalkboard a mathematical sentence.

e.g., $1.25 + 3 x $2.50 = ☐
ice cream cost   people   movie cost

Say, "How would you solve this problem?" Ask a student to calculate the answer. The student will probably say $1.25 + 3 = $4.50 X $2.50 = $11.00. Say, "If we calculate this problem in this way, we will think you do not have enough money but we will be wrong. Let's take another look at this problem." Write $1.25 + 3 X $2.50 = ☐

STRATEGY 2:
cont'd

Say, "The movie costs $2.50 for each person. To calculate the total cost of the movie, we multiply first.

e.g.,

$1.25 + (3 x $2.50) = □ .
$7.50

Then we add the cost of the ice cream. e.g.,$1.25+ $7.50 = $8.75. So, we do have enough money." Say that in order to avoid confusion, mathematicians devised an order of operations to insure one correct answer to a problem. Write $6 + 2^2 \times 3 - (9 \div 3) = \square$ on the chalkboard. Say that we calculate using this order.

e.g.,

$6 + 2^2 \times 3 - (9 \div 3) =$	1. Anything in parentheses.
$6 + 2^2 \times 3 - 3 =$	
$6 + 4 \times 3 - 3 =$	2. Exponents
$6 + 12 - 3 =$	3. Multiplication or Division (left to right)
$18 - 3 =$	4. Addition or Subtraction (left to right)
15	

Ask the students to solve $(6 \times 5) + 2^2 + 30 \div 2 =$ using order of operations. Ask a student to come to the chalkboard to solve the example. Reinforce by practicing solving problems using order of operations.

7.11.02

7.11.02.02 **determine whether a numerical expression is a phrase or a sentence**

STRATEGY 1: Write on the chalkboard.

(a) 7 x 3 + 4
(b) 6 + 4 = 10

Say, "What is the difference between a and b?" Elicit that (b) is a numerical sentence and that (a) is a numerical phrase. Ask a student what symbol will make a complete numerical sentence. Elicit that it is the equal sign (=). Repeat this with other problems.

e.g.,
(a) 8 + 2 x 5
(b) 9 + 1 x 4 = 36

Ask a student to give the definition of a numerical sentence. Elicit that a numerical sentence has the equal sign (=) in it.

STRATEGY 2: Write on the chalkboard, 8 + 3. Ask a student to come forward and compute the answer.
e.g., 8 + 3 = 11.

Repeat this using 4 x 5, 12 - 3, 18 ÷ 6. Say that 8 + 3 is a phrase because it is not complete and 8 + 3 = 11 is called a numerical sentence. Say the difference between a numerical phrase and a numerical sentence is the equal sign (=).Write several additional phrases on the chalkboard.
e.g., 16 ÷ 4, 18 x 3, 9634 + 8.

Ask a student to make complete sentences from the phrases you wrote on the chalkboard.

7.11.02

7.11.02.03 **translate a word sentence into a numerical sentence including parentheses**

STRATEGY 1: Write on the chalkboard.
e.g., Thirty-two divided by the sum of two and six is four.

Ask a student to check the answer to this problem orally, e.g., since the sum of 2 and 6 is 8 and 32 ÷ 8 = 4, the answer is correct. Ask another student to write the same problem on the chalkboard using mathematical symbols.
e.g., 32 ÷ 2 + 6 = 4.

Ask the student to follow the order of operations and solve this problem.
e.g.,

$$\begin{aligned} 32 \div 2 + 6 &= 4 \\ 16 + 6 &= 4 \\ 22 &= 4 \end{aligned}$$

Ask the students what could be added to the numerical translation to make the solution correct. Say, "Think back to the oral solution. What operation was performed first?" Elicit 2 + 6 = 8. Ask the students what symbol to add to make the sentence correct. Elicit parentheses and write, 32 ÷ (2 + 6) = 4. Say that the parenthesis tells us to add first and then divide. Repeat this procedure with other verbal translations.
e.g.,
If you add six to the product of five and three the result will be twenty-one. (5 x 3) + 6 = 21
Divide the difference of twenty and two by three so the result is six. (20 - 2) ÷ 3 = 6

STRATEGY 2: cont'd

Write a number using words on the chalkboard. e.g., twenty-four.

Ask the students to write a verbal sentence in which twenty-four is the solution. Ask the students to use any of the four basic operations: addition, subtraction, multiplication and division. Some sample problems may look like these.
e.g.,

(a) Four times six is twenty-four
(b) Eight times three is twenty-four.

Ask several students to write their problems on the chalkboard. Ask other students to translate the problems to numerical sentences. Say that the students may use more than one operation in a sentence.
e.g., Twelve added to the product of two and six is twenty-four.
Say that we can use parentheses in our problems. Repeat this procedure with other numbers.
e.g.,
sixty-four, twenty-seven, thirty-five.

7.11.02.04 determine whether a numerical statement is true or false

STRATEGY 1: Write a numerical sentence on the chalkboard.
e.g.,

8 + 4 = 2 x 6
4 x 3 + 2 = 15

7.11.02

STRATEGY 1:
cont'd

Ask a student to verify by computation on the chalkboard, whether the statement is true or false? Write 12 ÷ 2 + 4 = 10 on the chalkboard. Repeat this procedure by writing other true or false numerical sentences on the chalkboard. Say that a sentence is true if the numerical phrase on the left side of the equal sign has the same value as the numerical phrase on the right side of the equal sign. Ask the students to make up their own true and false numerical sentences. Ask the students to exchange sentences with their neighbors and to write true or false next to each statement. Ask the students to change each false statement to a true statement.

STRATEGY 2:

Prepare numerical sentences on oaktag cards.
e.g.,

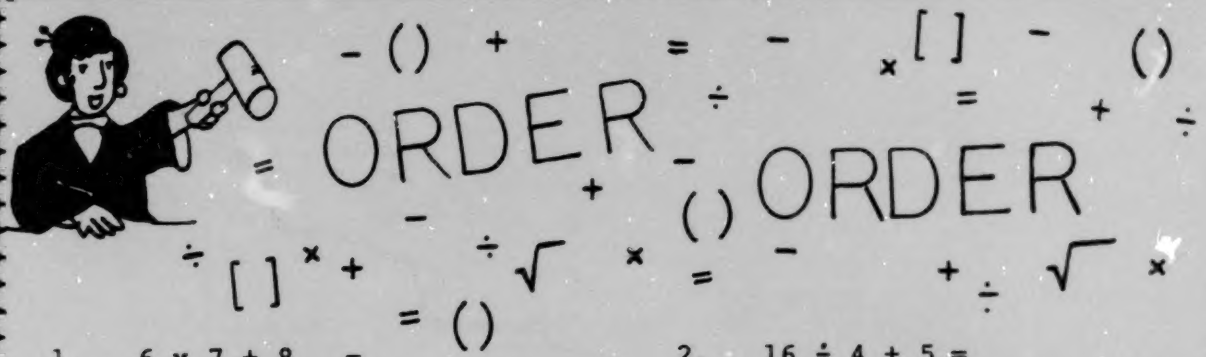
4 x 5 = 20
7 + 8 X 6 = 56
8 + 4 x 2 = 16
81 ÷ 3 - 5 = 22

Distribute one or more cards to each student. Ask the students to determine whether each sentence is true or false by computation. Ask the students to write their own true or false statements on the back of the cards. Ask the students to make the false statements true.

P.O. 7.11.02.01

NAME: ______________________ CLASS: ________ DATE: ________

Compute the following problems. Use the correct order of operations.



1. 6 x 7 + 8 =

2. 16 ÷ 4 + 5 =

3. 6 + 8 x 5 =

4. 14 - 4 ÷ 2 =

5. 6 x 8 - 12 ÷ 3 =

6. 48 = 6 x 7 + 3 x 2 =

7. 3(6+3) - 4 =

8. 25 + 3 (10-4) =

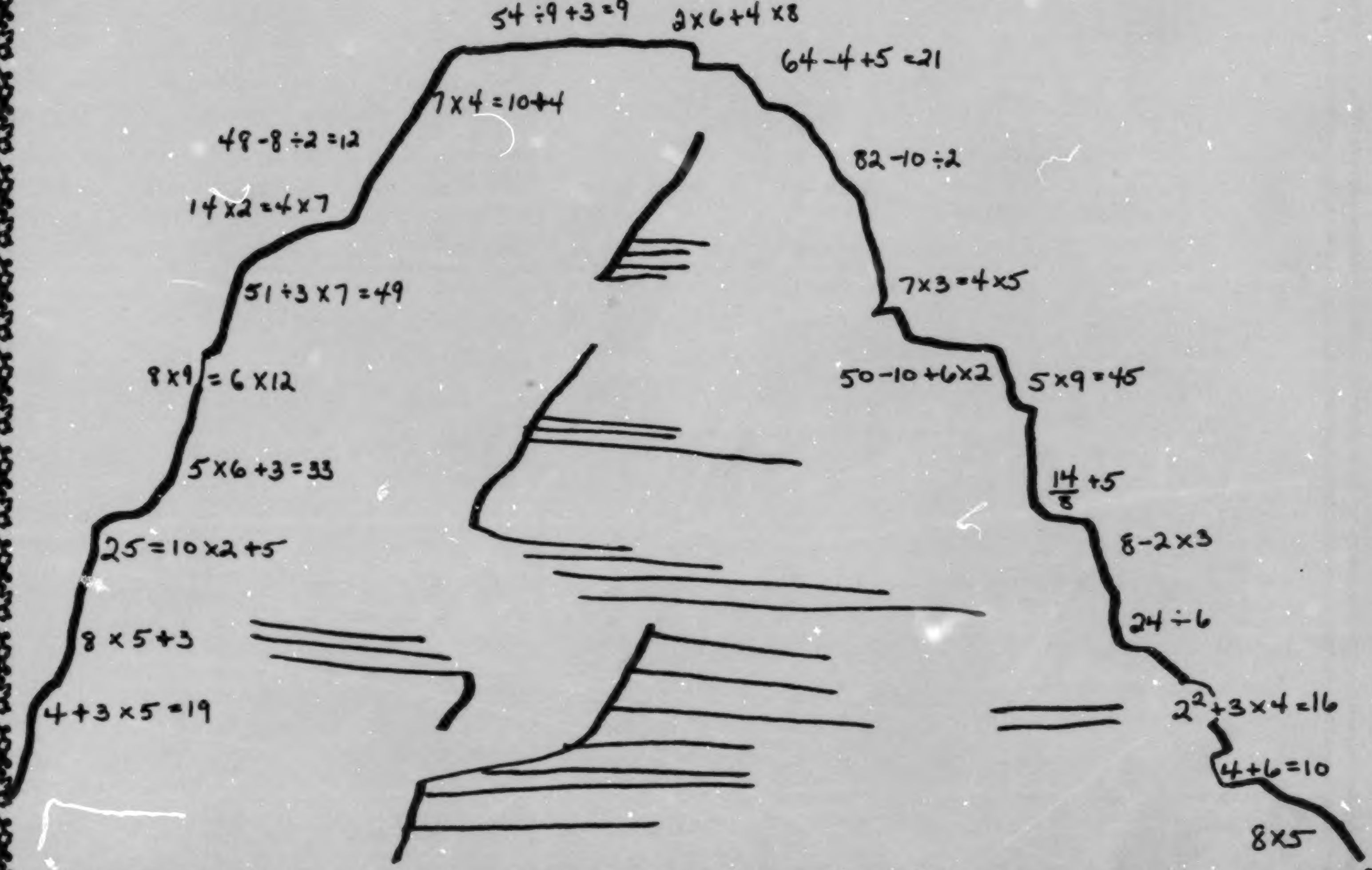
* 10 [4+ (7-1) x 5] =

P.O. 7.11.02.02

NAME: ____________________ CLASS: __________ DATE: __________

Write P for (phrase) or S for (sentence) as you climb up the mountain and as you go down.

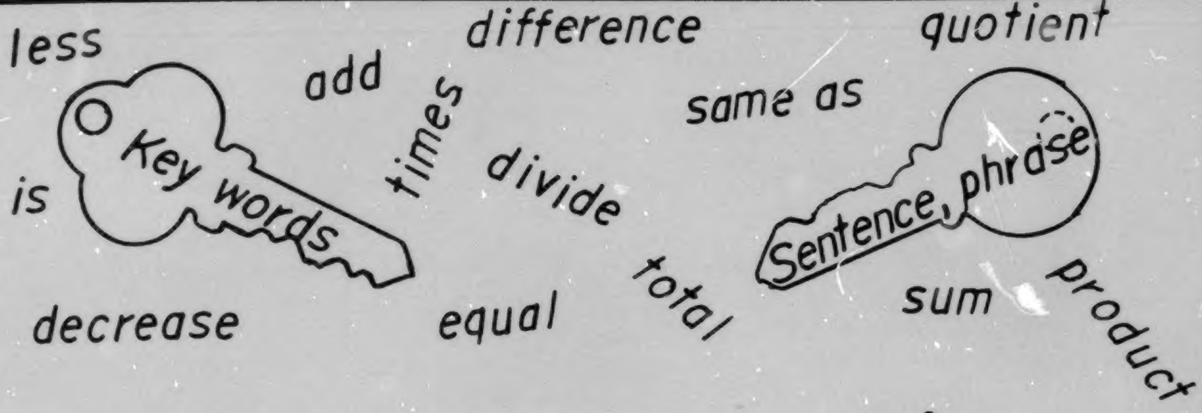
King of the Hill



P.O. 7.11.02.03

NAME:______________________________ **CLASS:**__________ **DATE:**__________

Draw a line from the words to the correct numerical expression.



1. Four increased by the product of two and one.
2. The sum of six and twelve is eighteen
3. The quotient of twenty-four and two decreased by five.
4. The sum of seven and fourteen is twenty-one.
5. Six times five is the same as sixty divided by two.
6. Twenty-six decreased by the quotient of eighteen and three.
7. Twelve to the second power divided by six.
8. Three times the sum of six and three is the same as three times nine.

(a) $12^2 \div 6$

(b) $7 + 14 = 21$

(c) $24 = 6 \times 4$

(d) $26 - 18 \div 3$

(e) $3(6+3) = 3 \times 9$

(f) $9 \times 2 = 18$

(g) $6 + 12 = 18$

(h) $6 \times 5 = 60 \div 2$

(i) $4 + 2 \times 1$

(j) $24 \div 2 - 5$

P.O. 7.11.02.04

NAME:____________________ CLASS:__________ DATE:________

Write true or false next to each numerical sentence.

1. 6 x 8 + 7 = 55 ____

2. 8 x 4 - 3 = 21 ____

3. 4 + 18 ÷ 2 = 11 ____

4. 17 = 3^2 + 8 ____

5. 5 + 8 x 4 = 37 ____

6. 14 + 2 = 4^2 ____

7. 12 x 2 - 24 ÷ 8 = 22 ____

8. 7 x 6 = 48 - 6 ____

9. 12 ÷ 2 = 1 + 2 + 3 ____

10. 4 + 7 x 3 = (4 + 7) x 3 ____

Change all the false statements so that they are true!

Can it be TRUE or Is it FALSE!

7.14.01 NUMBERS THROUGH BILLIONS

(ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

7.14.01.01 state the value of any digit in numbers through billions;

e.g.,

673,802,375

↓ 8 hundred thousands

↓ 70 millions

7.14.01.02 separate a number into its proper period;

e.g.,

6,	738,	409,	362
billions	millions	thousands	units

7.14.01.03 write the word name given a standard base 10 numeral;

e.g.,

6,743,891,602 =

six billion,
  seven hundred forty-three million,
    eight hundred ninety-one thousand,
      six hundred two

7.14.01.04 write the standard numeral given the word name.

e.g.,

four hundred thirty-two million,
eight hundred sixteen thousand,
ninety-seven = 432,816,097

7.14.01

VOCABULARY

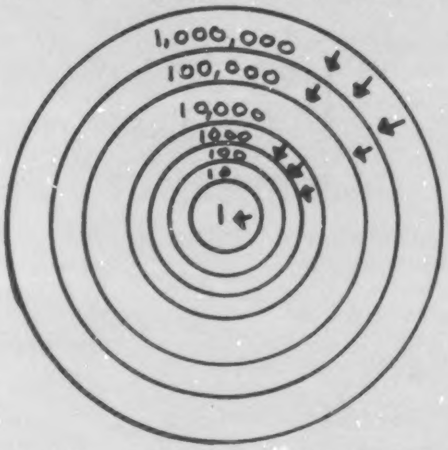
TERMS

place value
billions

GETTING STARTED

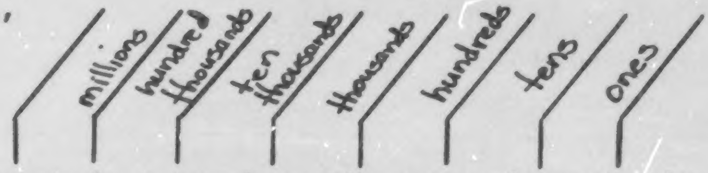
Draw on an overhead projector a dartboard transparency (see below) or copy it on the chalkboard. Place the darts as shown.

e.g.,

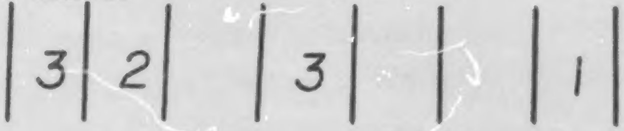


Draw a place value chart on the chalkboard.

e.g.,



Ask a student to place the score from the dartboard on the place value chart.



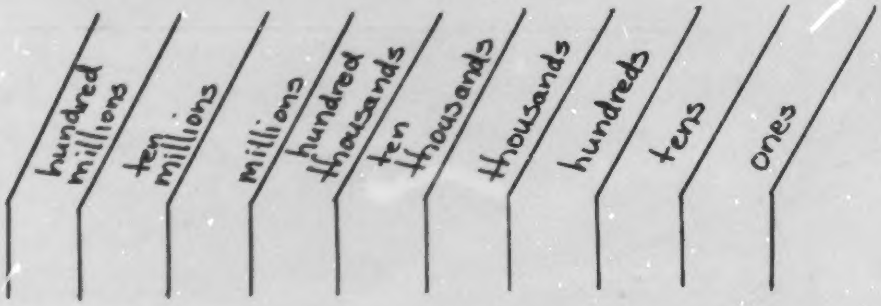
Rewrite the number on chalkboard, e.g., 3 2 _ 3 _ _ 1 .
Ask the students what numbers to enter in the blanks. Say to the students that we use zero as a placeholder. Repeat this exercise several times using different scores on the dartboard.

GETTING STARTED con't

Write these digits on the chalkboard.

① ⑨ ③ ⑦ ④ ⑧ ⑤ ② ⓪

Say, "What is the largest possible number you can make using these digits?" Write the place value headings on chalkboard.

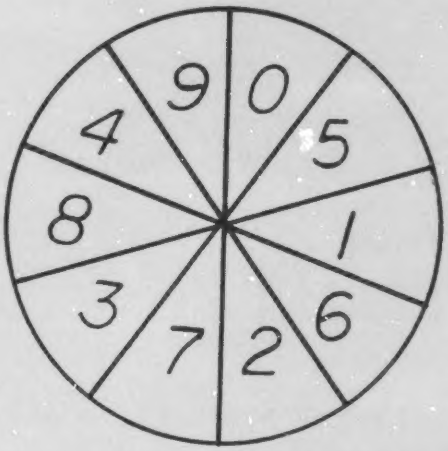


Ask a student to come to the chalkboard and write his/her answer in the chart. Ask the student to read his/her answer. Say, "What is the smallest possible number you can make using these digits?" Ask another student to write his/her answer in the chart. Repeat this procedure using nine other numbers.

7.14.01

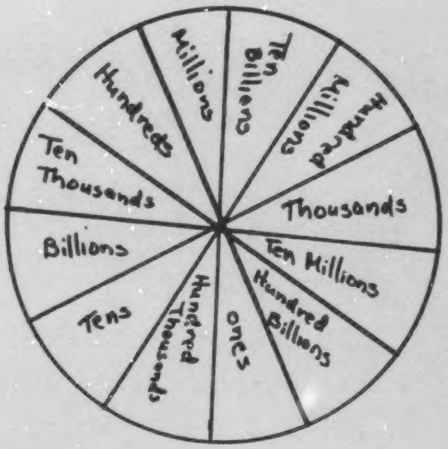
7.14.01.01 **state the value of any digit in numbers through billions**

STRATEGY 1: Prepare one digit-spinner, one place-value spinner and four playing boards for each group of four students.

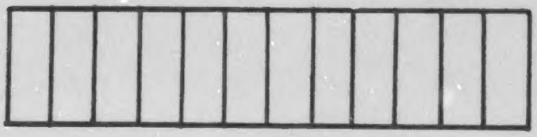


Digit-spinner

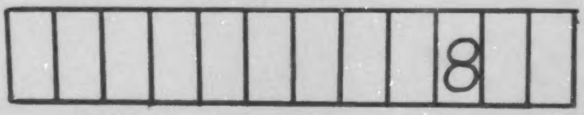
Place-value spinner



Playing board



Separate the class into groups of 4 players. Give each group a place-value spinner, a digits-spinner and a playing board. Say to the students, "Each player spins the digit-spinner. The player with the highest number begins. The first player spins the digit spinner and the place-value spinner. The player then writes the digit in the correct place on his/her playing board, e.g., if the player spins an "8" on the digits-spinner and a "hundreds" on place-value spinner the player writes 8 in the hundreds position."



7.14.01

STRATEGY 1: cont'd

Players take turns. If a player spins a place that has already been filled, the player loses his/her turn and play proceeds to the next player. When one player fills every space on his/her playing board, the game is over. The other players must then place a zero in each unfilled space of their playing boards. The players then compare their numbers. The player with the highest number wins.

STRATEGY 2: Write 623, 485, 677, 093 on the chalkboard. Ask a student to read the number. Point to any numeral in the number and say, "What is the place value of this digit? What is the face value of this digit?" Repeat this procedure by pointing to other numerals and asking their place and face value.

7.14.01.02 **separate a number into its proper period**

STRATEGY 1: Write a nine-digit number on the chalkboard without commas, e.g., 9 2 3 4 5 6 7 8 9. Ask a student to read the number. Ask another student to come to the chalkboard to put in commas. Say, "This is much easier to read." (If a stopwatch is available compare the times it takes students to read numbers with and without commas).

STRATEGY 2: Copy the following on the chalkboard.
e.g.,

— —, — — —, — — —, — — —

Ask students to come to chalkboard one by one and place the digits in the correct blanks.

40 million
90 billion
456 thousand
25 units

Repeat this procedure using other 11-digit numbers.

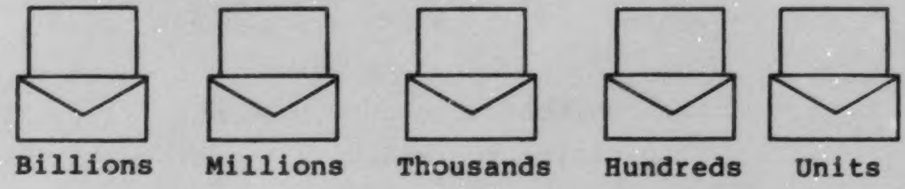
7.14.01

7.14.01.03 **write the word name given a standard base 10 numeral**

STRATEGY 1: Prepare enough digit cards (0-9) for an entire class. Separate students into groups of two to four. Say that each player will draw six to twelve cards and arrange them in the order in which they are drawn.

Say, "You will score 1 point for 6 cards, 2 points for 7 cards, 3 points for 8 cards, 4 points for 9 cards, 5 points for 10 cards, 6 points for 11 cards, 7 points for 12 cards, if the number is read correctly. If it is not read correctly, another player may read it to earn a score." Ask students to read the digit they have formed.

STRATEGY 2: Copy the following diagram on the chalkboard or an overhead transparency.



Ask a student to come to the chalkboard, place three digits in each cutout, and challenge another student to read his/her number. If the student reads the number correctly, he/she will then place other digits in the cutout. If the number is read incorrectly, the student should call on other students until a correct answer is given.

Repeat this procedure several times.

7.14.01.04 **write the standard numeral given the word name**

STRATEGY 1: Separate the students into five groups which are called billions, millions, thousands, units, and translators. Say, "A billions student will go to chalkboard and write a one, two or three digit number (in words) followed by the word billions. A million student will go to chalkboard and write a one, two or three digit number (in words) followed by the word millions." This procedure is repeated through units. A student from the "translators" will then go to the chalkboard and write the standard numeral for the number.

Repeat this procedure five times.

STRATEGY 2: Say, "You have a choice of making up a number in the thousands, millions or billions. You will score 1 point for thousands, 2 points for millions, 3 points for billions. You will say your number and call upon another student to write it on the chalkboard. If the student writes it correctly, you will double your points. If he/she writes it incorrectly you will lose your points for that number. The student who writes it correctly will then make up a new number."

Repeat this procedure several times.

P.O. 7.14.01.01

NAME:____ CLASS:________ DATE:________

Find the letter corresponding to the underlined digit which is equal in value to the numeral. Write the letter above the numeral.

EXAMPLE: 50,000 is the value of the underlined digit in 342,150,896, which is represented by R.

WHO WAS THIS FAMOUS ATHLETE?

This famous football player racked up one of the most remarkable individual game records in 1924, playing against an unbeaten University of Michigan team. He ran the opening kick off back 95 yards for a touch down. Then, just a couple of minutes later he went 67 yards around the end for a second touch down. In that same first quarter he ran 56 yards for a third touchdown. Altogether by the end of the game he had run for four touchdowns and passed for two more. After college he played professional football for the Chicago Bears.

R

40,000,000 5,000,000,000 50,000 5,000 500,000 500,000,000

50,000 4,000,000 500,000,000 900,000,000,000 50,000

5,000,000,000 90,000,000 900,000,000,000 4,000,000 50

5,000,000,000 90,000 900 90,000,000 5000 50 90,000,000

5,000,000,000 90,000 40,000 40,000,000 4,000,000

900,000,000,000 5,000,000,000 500,000 500,000 5000 40

400 90,000,000 900,000,000,000 900,000,000,000 40,000,000

5,000 90,000 40,000

P.O. 7.14.01.01 cont'd

NAME:________________ CLASS:__________ DATE:________

A	B	C	D	E
10$\underline{5}$,869,324,040	$\underline{4}$21,986,357	689,$\underline{4}$12,375	$\underline{5}$01,698,423	12$\underline{4}$,869,537
F	G	H	I	J
708,243,1$\underline{9}$6	$\underline{9}$81,241,682,401	2$\underline{4}$1,689,735	679,521,$\underline{4}$38	1$\underline{5}$0,689,324
K	L	M	N	O
780,423,$\underline{9}$16	689,$\underline{5}$10,423	324,869,$\underline{5}$10	8$\underline{9}$0,762,341	432,01$\underline{5}$,698
P	R	S	T	W
967,215,8$\underline{4}$3	342,1$\underline{5}$0,896	324,6$\underline{9}$1,708	896,1$\underline{4}$2,573	968,423,1$\underline{5}$0

P.O. 7.14.01.02

NAME:______________________________CLASS:_________DATE:_______

Write each with commas.

SEPARATE A NUMBER INTO ITS PERIODS

625400	90642	352180750	820000
59362	40085	9955110500	26543218
5000000	92643	4524680900	9020
6523841	420000	9630500520	4444432

P.O. 7.14.01.03-04

NAME: ________________________ CLASS: __________ DATE: ________

Match the number to the word name.

B	(1)	70,400,000	A	seventy billion, four hundred thousand
___	(2)	740,000,000,000	B	seventy million, four hundred thousand
___	(3)	704,000,000	C	seventy billion, four hundred million
___	(4)	74,000,000	D	seven hundred four billion, four hundred million
___	(5)	70,400,000,000	E	seventy billion, forty million
___	(6)	70,000,400,000	F	seven billion, forty million
___	(7)	7,040,000,000	G	seven hundred forty billion
___	(8)	700,000,040,000	H	seventy - four million
___	(9)	704,400,000,000	I	seven hundred billion, forty thousand
___	(10)	70,040,000,000	J	seven hundred four million

7.02.01 ADDITION OF WHOLE NUMBERS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.02.01.01A compute the sums of whole numbers and check; e.g.,

$$\begin{array}{r} 64{,}606{,}732{,}875 \\ +\ \underline{14{,}893{,}806{,}775} \\ 79{,}500{,}539{,}650 \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 79{,}500{,}539{,}650 \\ -\ \underline{14{,}893{,}806{,}775} \\ 64{,}606{,}732{,}875 \end{array}$$

7.02.01.02A estimate by rounding addends to the largest possible place; e.g.,

$$\begin{array}{rcr} 64{,}606{,}732{,}875 & \text{rounds to} & 60{,}000{,}000{,}000 \\ +\ \underline{14{,}893{,}806{,}775} & \text{rounds to} & \underline{10{,}000{,}000{,}000} \\ 79{,}500{,}539{,}650 & & 70{,}000{,}000{,}000 \end{array}$$

7.02.01.03A solve verbal problems related to skills learned in this module. e.g.,

Your auto insurance cost includes $218 for collision, $219 for bodily injury liability and $72 for property damage. If you already have saved $600 for insurance, will you have enough to cover the total cost?

$$\begin{array}{r} \$218 \\ 219 \\ +\ \underline{\ \ 72} \\ \$509 \end{array} \qquad \begin{array}{r} \$600 \\ \underline{-509} \\ 91 \end{array}$$

7.02.01

VOCABULARY

TERMS

palindrome

GETTING STARTED

Say that palindromes are words or numbers that read the same forward and backward, e.g., mom, toot, madam, 6,776, 9,865,689. Write the following on the chalkboard.

> A palindrome can be formed from a number that is not a palindrome by following these steps.

Step 1: Pick a number 34
Step 2: Reverse the digits of the number +43
Step 3: Add the number 77

Ask the students to follow this procedure using as beginning numbers: 23, 38, 65 and 415. Say to the students that for most numbers, a palindrome cannot be formed in three steps. Do the following example on the chalkboard.

Step 1: Pick a number 174
Step 2: Reverse the digits +471
Step 3: Add 645
Step 4: Reverse +546
Step 5: Add 1191
Step 6: Reverse +1911
Step 7: Add 3102
Step 8: Reverse +2013
Step 9: Add 5115 A Palindrome!

Ask the students to find the palindromes that result from the following starting numbers.
e.g., 851, 561, 1709.

Ask the students to put away all pencils, pens, paper, notebooks. Say, "This will be a contest of how well you can add in your head.

GETTING STARTED cont'd

To add 43 and 29 for example, you might think 40 + 20 = 60 and 3 + 9 = 12 so 60 + 12 = 72." Divide the students into two teams. Give one point for each correct answer. Some suggested examples to be given orally are: 23 + 48, 57 + 29, 68 + 54, 48 + 21. More than two addends can be used, depending on the capability of the students.

7.02.01

7.02.01.01 **compute the sums of whole numbers and check**

STRATEGY 1: Ask the students to find the missing digits in the following examples and check by subtracting.
e.g.,

$$\begin{array}{r} 3\ \boxed{}\ 5\ 7 \\ +1\ 9\ \boxed{}\boxed{} \\ \hline \boxed{}\ 5\ 5\ 2 \\ -1\ 9\ \boxed{}\boxed{} \\ \hline 3\ \boxed{}\ 5\ 7 \end{array} \quad \longrightarrow \quad \begin{array}{r} 3\ 5\ 5\ 7 \\ +1\ 9\ 9\ 5 \\ \hline 5\ 5\ 5\ 2 \\ -1\ 9\ 9\ 5 \\ \hline 3\ 5\ 5\ 7 \end{array}$$

Say that the subtraction check can help to find the missing digits. Repeat this procedure using other examples.

STRATEGY 2: Place the following addition problems on the chalkboard.

$$\begin{array}{r} 59 \\ +68 \\ \hline \end{array} \qquad \begin{array}{r} 269 \\ +\ 78 \\ \hline \end{array} \qquad \begin{array}{r} 568 \\ +343 \\ \hline \end{array} \qquad \begin{array}{r} 8762 \\ +3409 \\ \hline \end{array}$$

$$\begin{array}{r} 676 \\ 394 \\ +528 \\ \hline \end{array} \qquad \begin{array}{r} 3,857 \\ 296 \\ +17,847 \\ \hline \end{array} \qquad \begin{array}{r} 697,283 \\ +208,865 \\ \hline \end{array}$$

$$\begin{array}{r} 24,082,615 \\ +15,704,986 \\ \hline \end{array} \qquad 97 + 546 + 8023 = \boxed{}$$

$$101,402 + 68 + 3,406,735 = \boxed{}$$

Discuss with students ways of checking the answer, e.g., if two addends are given, check the sum by subtracting the second addend from the sum

STRATEGY 2: cont'd

resulting in first addend; if more than two addends are given the sum can be checked by reversing the direction of addition.

e.g.,

$$\begin{array}{r} 54 \\ +68 \\ \underline{\;76} \end{array} \qquad \begin{array}{r} 54 \\ +68 \\ \underline{+76} \end{array}$$

7.02.01.02 **estimate by rounding addends to the largest possible place**

STRATEGY 1: Divide the students into two teams. The first person on Team I writes a 4- to 9-digit number on the chalkboard. The first person on Team II rounds the number to the highest place value and writes the rounded number on the chalkboard.

e.g.,

NUMBER	ROUNDED TO THE LARGEST PLACE
45,678	50,000

The next person on Team II writes another number on the chalkboard and the next member of Team I writes the rounded number underneath. The third member of Team II adds the two rounded numbers and records the sum.

e.g.,

$$\begin{array}{r} 45{,}678 \\ \underline{+310{,}205} \end{array} \qquad \begin{array}{r} 50{,}000 \\ \underline{+300{,}000} \\ 350{,}000 \end{array}$$

Repeat this procedure several times.

STRATEGY 2: Prepare an assortment of newspaper advertisements or catalog pages that include listings of items with their prices. Ask the students to choose four or more items they would like to buy. Ask them to

STRATEGY 2:
cont'd

round the prices to the highest dollar place value and add the estimated values. Then ask them to calculate the actual sums to find out how close their estimates were.

7.02.01.03 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Distribute to the students newspaper advertisements or supermarket flyers advertising "Specials of the Week." Ask them to make up word problems using the information on the flyers. Ask a student to write his/her problem on the chalkboard and select a classmate to solve the problem.
Continue eliciting other students problems.

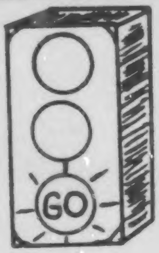
STRATEGY 2: On a set of 3" by 5" index cards write one of these words or phrases on each card: in all, total, sum, plus, add, spend, cost. Divide the students into two teams. The first person in Team I selects a card and makes up a simple word problem which includes the word or phrase on the card. The first person in Team II tries to solve the problem. Team I earns a point for the made-up problem; Team II earns a point for the solution.

Play and alternate, having the second person in Team II making up the problem and the second person in Team I trying to solve it.

P.O. 7.02.01.01

NAME: ______________________ **CLASS:** __________ **DATE:** __________

Do the exercises as indicated.



1. 338
+259

2. 867
+290

3. 805
+367

4. 12,824
+ 1,302

5. 361.09
+558.80

6. 657
+269

7. 983
+ 124

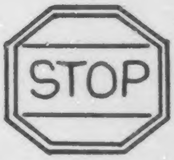
8. 5,856
+1,868

9. 1,748
+ 292

10. $782.28
+ 63.84

11. 2,411 + 13,570 + 924

12. 135,906 + 3,058 + 1,457



40

P.O. 7.02.01.02

NAME: ______________________ CLASS: __________ DATE: ________

Estimate each sum. Then add.

	Estimate		Estimate		Estimate
373 +499 872	900	4235 +3845		736,495 +206,667	
34,281 +98,311		775,100 +245,209		85,125 +20,999	
9475 +1009		5762 +3048		67,113 +25,246	
2815 382 1134 + 899		67,348 1,255 72,118 + 3,485		67,777 95,028 13,154 +26,340	

P.O. 7.02.01.03

NAME:____CLASS:________DATE:_______

Use the chart to solve the following problems.

Activity	Calories Used in 1 Hour
Football	558
House cleaning	256
Jogging	567
Shoveling snow	751
Studying	68
Swimming	648
Tennis	444
Walking	296
Watching T.V.	48

1. Jane's exercise schedule on Monday and Wednesdays included one hour of walking and two hours of swimming. How many calories did Jane use in these activities?

2. John wanted to lose weight. Five days a week, he played tennis two hours per day; three days a week he walked, one hour per day. How many calories did John use in one week?

3. What two activities would caused you to lose closest to 1,000 calories in two hours?

4. Which activities on the chart did you do yesterday? Figure the total calories you used in these activities.

5. Aaron studied for two hours and swam for two hours. What else did Aaron do if he used a total of 1728 calories?

P.O. 7.02.01.03

NAME: ______________________________ CLASS: __________ DATE: ________

Read each problem. Decide which operation you will use. Estimate your answer. Solve the problem.

WORK SPACE

1. The distance from Berlin to Paris is 1094 Km. From Paris to Venice it is 1145 Km and from Venice to Madrid it is 1955 Km. If Pat and Mike travel from Berlin to Paris, from Paris to Venice and from Venice to Rome how many Km will they travel?

   What operation?
   Estimate answer.

2. The population of Home Town, USA is 179,323,108. The population of Uptown, USA is 205,306,449. What is the combined population of both towns?

   What operation?
   Estimate answer.

3. In 1955, there were about 175,362,458 math teachers. The number may increase by 26,000,000 in 1990. How many math teachers will there be in 1990?

   What operation?
   Estimate answer.

7.01.01 PALINDROMES

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.01.01.01A identify a palindrome;
e.g.,

51315

MADAM

7.01.01.02B find a palindrome by repeated additions;
e.g.,

$$\begin{array}{rl} 86 & \\ +\ \underline{68} & \\ 154 & \text{no palindrome} \\ +\ \underline{451} & \\ 605 & \text{no palindrome} \\ +\ \underline{506} & \\ 1111 & \text{palindrome} \end{array}$$

7.01.01.03B classify a palindrome.
e.g.,

86 is a class 3 number because 3 additions are necessary.

7.01.01

VOCABULARY

TERMS

palindrome
class number

GETTING STARTED

Write the following numbers on the chalkboard, 1, 5, 7, 9, 7, 5, 1. Ask a student to identify the pattern (add 2 three times, then subtract 2 three times). Ask another student to design a pattern or sequence of his/her own and come to the chalkboard to write the pattern. Allow the student to ask the class to discover the pattern. Repeat this procedure several times using different student sequences.

Write these problems on the chalkboard.

```
  45        99        198
 +54       +99       +891
 (99)     (198)     (1089)
```

Ask three students to come to the chalkboard to solve one problem each. Say, "Do you see a pattern to the problems and sums written on the chalkboard?" The first addend is reversed to form the second addend; the sum is always the next addend.

Repeat this procedure using the following examples.

```
 13     44
+31    +44
```

7.01.01

7.01.01.01 **identify a palindrome**

STRATEGY 1: Write on the chalkboard the following list of words: madam, noon, dad, toot, radar, peep. Say, "Do you see anything special about these words? (They spell the same word forward and backward). Does anyone know what we call this special kind of word? (palindrome) Numbers can also be palindromes. What do you think a numerical palindrome would look like? (1221)" Ask several students to think of an original palindrome and write each one on the chalkboard.

STRATEGY 2: Distribute the following cards to five students $\boxed{3}$ $\boxed{2}$ $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ and ask them to stand in this order at the front of the room. Ask a student to read the number to the class. Ask the class if they see anything unusual about the number (The order of the digits in the number are the same forward and backward). Ask the students to change places, $\boxed{3}$ $\boxed{2}$ $\boxed{1}$ $\boxed{2}$ $\boxed{3}$
Elicit if the number was changed. Say, "Can anyone think of another number like this one?" Elicit several answers and write the other numbers on the chalkboard. Say, "What do we call a number that is exactly the same whether we read it forward or backward?" (palindrome). Write the word and definition on the chalkboard. Explain that a palindrome may be a word, number, phrase or sentence. Ask the students if they can think of any alphabetic palindromes (nun, pop, mom, etc.) Say, "Can you think of a palindrome sentence?" (Too hot to hoot.)

7.01.01

7.01.01.02 **find a palindrome by repeated additions**

STRATEGY 1: Say, "We can take a number that is not a palindrome and change it into a palindrome by a special process." Elicit from a student a 2-digit number, e.g., 13, 68. Write the number on the chalkboard. Say, "What number will we get if we reverse the digits? (31, 86) Now add the reversed number to the original number."

e.g., $\begin{array}{r} 13 \\ \underline{+31} \end{array}$

Write the example on the chalkboard. Ask a student for the sum. Examine the sum. Elicit if it is a palindrome. Continue the procedure until the sum is a palindrome.

$$\begin{array}{rl} 13 & \\ \underline{+31} & \\ 44 & \text{palindrome} \end{array} \qquad \begin{array}{rl} 68 & \\ \underline{+86} & \\ 154 & \text{no palindrome} \\ \underline{+451} & \\ 605 & \text{no palindrome} \\ \underline{+506} & \\ 1111 & \text{palindrome} \end{array}$$

(Students may want to use a calculator if the additions get very large.)

STRATEGY 2: Prepare a deck of cards eliminating the picture cards. Deal two cards to one student (1 (ace) ♡ , 5 ◇). Ask the student to come to the chalkboard and write the number made from the cards (15). Ask the student to reverse the cards thereby reversing the digits (51). Write this under the first number and add.

e.g., $\begin{array}{r} 15 \\ \underline{+51} \\ 66 \end{array}$

Say, "Did we get a palindrome?" (Yes) Explain that this doesn't always happen on the first addition.

STRATEGY 2: cont'd

e.g.,

$$\begin{array}{r} 49 \\ +94 \\ \hline 143 \end{array}$$

In such a case we have to repeat the procedure with the new sum.

e.g.,

$$\begin{array}{r} 143 \\ +341 \\ \hline 484 \end{array}$$

Say, "We are going to play a palindrome game. I will deal cards to each of you. The first student to get a palindrome from the 2 cards will stand, come to the chalkboard, and write his/her additions. If you are correct you will get a point. We will continue the game until someone gets 3 points."

Additional decks of cards may be needed depending on the size of the class.

Play the game until someone wins.

7.01.01.03 **classify a palindrome**

STRATEGY 1: Write 28 on the chalkboard. Ask a student to come to the chalkboard and find a palindrome using repeated addition.

e.g.,

$$\begin{array}{r} 28 \\ +82 \\ \hline 110 \\ +011 \\ \hline 121 \end{array}$$

Say, "How many additions did we do to arrive at the palindrome 121? (2) Since we did 2 additions, we call 28 a class 2 number." Repeat this procedure using 49.

STRATEGY 2: Divide the class into groups of five students. To each group distribute numbered graph paper, one number in each box, from 0 through 99 and crayons. Ask each group to find all the palindromes for all the numbers on the graph paper starting at number 10. If the number is already a palindrome, e.g., 11, 22, color the number's box green. If it takes only 1 addition, e.g., 12, 42, color the number's box red, 2 additions color blue, e.g., 28, 3 additions color yellow, 4 additions color orange, greater than 4 additions color purple. Say, "Divide the work so that some students add and some students record." When the groups are finished say, "Do you see any patterns on the graph paper? (Multiples of 11 are already palindromes; when the sum of the digits is less than 10, the number will be a 1-addition palindrome.) We categorize each number by the number of additions we must do to get a palindrome. Therefore, 42 is called a class 2 number while 68 is called a class 3 number." Ask students to classify several numbers.

P.O. 7.01.01.01-.02

NAME:________________________________ CLASS:__________ DATE:________

I. Circle each palindrome
II. Find the palindrome by repeated addition

I.

66
2345
1111
peep
people

too hot to hoot
4,632,364
madam
8008
otto

14,63,421
egg
12,344,321
museum
level

II.

(1) 46
(2) 23
(3) 73
(4) 81
(5) 40
(6) 252

P.O. 7.01.01.03

NAME:______________________________CLASS:________DATE:______

Color in the chart: palindrome: green, 1 addition: red 2 - addition: blue, 3 - addition: yellow, 4 - addition: orange more than 4 addition: purple.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

7.03.01 SUBTRACTION OF LARGE NUMBERS

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.03.01.01A subtract large numbers and check;

e.g.,

$$
\begin{array}{rrrrrrrrrrrr}
 & \overset{5}{\cancel{6}} & \overset{16}{\cancel{7}}, & 6 & 7 & 5, & \overset{5}{\cancel{6}} & \overset{9}{\cancel{0}} & \overset{11}{\cancel{2}}, & \overset{15}{\cancel{5}} & 9 & 9 \\
- & 1 & 8, & 9 & 3 & 2, & 1 & 1 & 8, & 7 & 5 & 3 \\
\hline
 & 4 & 8, & 7 & 4 & 3, & 4 & 8 & 3, & 8 & 4 & 6
\end{array}
$$

check

$$
\begin{array}{rr}
 & 48{,}743{,}483{,}846 \\
+ & 18{,}932{,}118{,}753 \\
\hline
 & 67{,}675{,}602{,}599
\end{array}
$$

7.03.01.02A estimate differences by rounding to the largest possible place;

e.g.,

$$
\begin{array}{rrcrr}
 & 6{,}485 & \text{rounds to} & & 6{,}500 \\
- & \underline{3{,}752} & \text{rounds to} & - & \underline{3{,}800} \\
 & & & & 2{,}700
\end{array}
$$

7.03.01.03A solve verbal problems related to skills learned in this module.

e.g., The distance from Halifax to Brooke County is 4497 km. The Frank family has already driven 177 km. How many more days must they drive if they cover 360 km per day?

$$
\begin{array}{r}
4497 \\
\underline{-177} \\
4320 \text{ km}
\end{array}
\qquad
\begin{array}{r}
12 \text{ days} \\
360\overline{)4320} \\
\underline{360} \\
720 \\
\underline{720} \\
0
\end{array}
$$

71

7.03.01

VOCABULARY

TERMS

difference
subtrahend
minuend
regroup (exchange)

GETTING STARTED

Write on the chalkboard, $\begin{array}{r} 84. \\ \underline{-53} \end{array}$ Ask the students to name the parts of the example.

e.g., $\begin{array}{rl} 84 & \text{minuend} \\ \underline{-53} & \underline{\text{subtrahend}} \end{array}$

Ask the students to solve the example. Say, "How do we check a subtraction example?" Elicit that since subtraction is the opposite of addition, we check subtraction by adding the difference to the subtrahend and the result should be the minuend.

e.g.,

$\begin{array}{rl} 84 & \text{subtrahend} \\ \underline{-53} & \text{minuend} \\ 31 & \text{difference} \end{array}$

check

$\begin{array}{rl} 31 & \text{difference} \\ +\underline{53} & \text{subtrahend} \\ 84 & \text{minuend} \end{array}$

Repeat the procedure using the following examples.

1. $\begin{array}{r} 527 \\ -\underline{254} \end{array}$

2. $\begin{array}{r} 730 \\ -\underline{562} \end{array}$

3. 482 - 167 = ☐

4. subtract 47 from 81.

Write 43,654 on the chalkboard. Say, "What is this number rounded to the nearest ten? hundred? thousand?" Elicit 43,650, 43,700, 44,000. Say, "What numbers help us to determine how to round?" Elicit that the number to the right of the place helps us to round. If the number is 0 through 4, we round to the lower ten, hundred or thousand, and if the number is 5 through 9, we round to the higher ten, hundred or thousand.

Repeat this procedure using 84,580, 7,699, 90,235 and 20,086.

7.03.01

7.03.01.01 **subtract large numbers and check**

STRATEGY 1: Write the following on the chalkboard.

> The Voyager VI spacecraft has traveled 6,586,452,112 miles. Voyager V spacecraft has traveled 4,277,384,502 miles. How much farther has the Voyager VI spacecraft traveled than the Voyager V spacecraft?"

Ask the students how to solve this example. Elicit that we need to find the difference between the two distances to determine how much further Voyager VI traveled. We must subtract.

e.g.,

$$\begin{array}{r} 6{,}586{,}452{,}112 \\ \underline{-4{,}277{,}384{,}502} \end{array}$$

Elicit that digits must be aligned correctly in the columns so that 2,309,067,610 + 4,277,384,502 = 6,586,452,112.

Repeat this procedure with other subtraction problems with large numbers.

STRATEGY 2: Write the following on the chalkboard.

> The planes of Titanic Airlines logged 62,817,274,905 miles this year. Last year they logged 45,723,605,724 miles. How many more miles were logged this year than last year?

Say, "What operation would you use to solve this problem?" Elicit that we need to find the difference between the two mileages to determine how many more miles were logged this year than last year. We must subtract.

e.g.,

$$\begin{array}{r} 62{,}817{,}274{,}905 \\ \underline{-45{,}723{,}605{,}724} \end{array}$$

Say that the digits must be aligned correctly in the columns. Say, "We can extend the methods of

STRATEGY 2: cont'd

subtraction that we used for smaller numbers to these larger numbers." Ask the students to subtract, and elicit from volunteers the following steps involving regrouping in performing the subtraction.

Step 1:

$$\begin{array}{r} 62,817,274,\overset{8}{\cancel{9}}\overset{10}{\cancel{0}}5 \\ -45,723,605,724 \\ \hline 181 \end{array}$$

Step 2:

$$\begin{array}{r} 62,817,2\overset{6}{\cancel{7}}\overset{14}{\cancel{4}},\overset{8}{\cancel{9}}\overset{10}{\cancel{0}}5 \\ -45,723,605,724 \\ \hline 69,181 \end{array}$$

Step 3

$$\begin{array}{r} 62,81\overset{6}{\cancel{7}},\overset{12}{\cancel{2}}\overset{6}{\cancel{7}}\overset{14}{\cancel{4}},\overset{8}{\cancel{9}}\overset{10}{\cancel{0}}5 \\ -45,723,605,724 \\ \hline 3,669,181 \end{array}$$

Step 4:

$$\begin{array}{r} 62,\overset{7}{\cancel{8}}\overset{11}{\cancel{1}}\overset{6}{\cancel{7}},\overset{12}{\cancel{2}}\overset{6}{\cancel{7}}\overset{14}{\cancel{4}},\overset{8}{\cancel{9}}\overset{10}{\cancel{0}}5 \\ -45,723,605,724 \\ \hline 093,669,181 \end{array}$$

Step 5

$$\begin{array}{r} \overset{5}{\cancel{6}}\overset{12}{\cancel{2}},\overset{7}{\cancel{8}}\overset{11}{\cancel{1}}\overset{6}{\cancel{7}},\overset{12}{\cancel{2}}\overset{6}{\cancel{7}}\overset{14}{\cancel{4}},\overset{8}{\cancel{9}}\overset{10}{\cancel{0}}5 \\ -45,723,605,724 \\ \hline 17,093,669,181 \end{array}$$

The planes logged 17,093,669,181 more miles this year than last year. Ask the students to check this to confirm that the method used was valid. Elicit that the difference plus the subtrahend should equal the minuend.

e.g.,

17,093,669,181 + 45,723,605,724 = 62,817,274,905.

Repeat this procedure using other subtraction problems.

7.03.01.02 **estimate differences by rounding to the largest possible place**

STRATEGY 1: Write the following on the chalkboard.

Mr. Jones earned $72,468 this year. Last year he earned $59,954. How much more did he earn this year than last year?

7.03.01

STRATEGY 1: cont'd

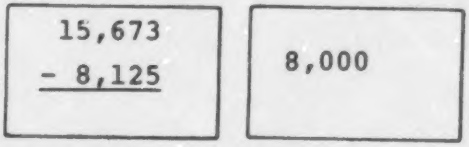
Say, "What operation would you use to solve the problem?" Elicit that subtraction is necessary to answer the problem. Ask a student to quickly give you an estimate to the largest possible place for the difference by rounding each number and then subtracting. Elicit that 72,468 rounds to 70,000 and that 59,954 rounds to 60,000 and that the result is 10,000. Say that rounding to the largest place gives us the roughest estimate, but is the easiest to compute. Ask students to determine if this is a good estimate. Elicit that 72,468 - 59,954 = 12,514, therefore, the estimated result was a good approximation.

Repeat this procedure using other subtraction problems.

STRATEGY 2:

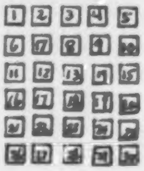
Prepare 30 cards, 15 cards having subtraction examples on them and the other 15 having the estimated answers.

e.g.,



Write a number on the back of each card. Tape the cards to the chalkboard in rows of five.

e.g.,



Ask a student to turn over two cards. If they match, the student gets another turn.

Repeat this procedure until all the pairs have been revealed.

7.03.01

7.03.01.03 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following on the chalkboard.

> A Burger-King restaurant ordered 37,500 hamburgers one month. If 28,412 were already received, how many more hamburgers are still due?

Ask the students which operation should be used to solve the problem. Elicit that subtraction is the operation to be used. Ask the students first to estimate the difference by rounding.

Repeat this procedure using other verbal problems.

STRATEGY 2; Write problems on cards and distribute one card to each student.

e.g.,

> In 1982, there were 16,894 finishers in the New York Marathon Race. In 1978, there were 8,598 finishers. How many more finished the race in 1982 than in 1978?

Say that we must find the difference to solve the problem and that before we subtract, we should estimate all the answers. Ask the students to solve the problems and check them in their notebooks. Ask the students to exchange cards with their neighbors as another check to each problem.

P.O. 7.03.01.01

NAME:______________________________ CLASS:_________ DATE:________

Subtract and check.

Example: 986,453,905
-160,728,903
825,725,002

check: 825,725,002
160,728,903
986,453,905

1. 45,560,398,721 -39,952,347,850	2. 235,999,002 -208,879,561	3. 3,288,769,423 - 921,567,018
4. 6,888,940 -5,999,760	5. 906,742,378 -111,238,379	6. 2,911,238,794 - 813,791,684

P.O. 7.03.01.02

NAME: ______________________ CLASS: ________ DATE: ________

Estimate differences by rounding to the largest possible place and then subtract.

Example: 42,951 rounding to 40,000
-29,328 rounding to -30,000
10,000 (estimate)

$$\begin{array}{r} 42{,}951 \\ -\underline{29{,}328} \end{array} \qquad \begin{array}{r} 40{,}000 \\ -\underline{30{,}000} \\ 10{,}000 \end{array} \text{ (estimate)}$$

1. $\begin{array}{r} 6943 \\ -\underline{2229} \end{array}$

2. $\begin{array}{r} 989{,}120 \\ -\underline{720{,}964} \end{array}$

3. $\begin{array}{r} 36{,}875 \\ -\underline{27{,}403} \end{array}$

4. $\begin{array}{r} 72{,}946{,}120 \\ -\underline{59{,}621{,}436} \end{array}$

*5. $\begin{array}{r} 1{,}256{,}420 \\ -\underline{834{,}589} \end{array}$

*6. $\begin{array}{r} 29{,}543 \\ -\underline{9{,}275} \end{array}$

P.O. 7.03.01.03

NAME:____CLASS:________DATE:_______

Solve.

1. Henry bought land for $1,923. Five years later he sold the land for $[illegible],592. How much profit did he make?

2. The Miller Family bought a house for $75,594. If they made a down payment of $10,888, how much more remains to be paid?

3. In the last election in a small town, 42,817 voters went to the polls out of a possible 186,217 registered voters. How many registered voters did not vote?

4. The population of Averagetown, Florida, increased from 283,618 to 1,017,921 in the last 10 years. Find the increase in population.

7.13.01 SOLVING A SIMPLER BUT EQUIVALENT PROBLEM

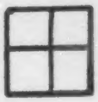
(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.13.01.01A use models/chart to simplify a problem; e.g.,

How many squares of all sizes are in an 8x8 checkerboard?



hint: 5 squares

total squares

(1x1) = $\boxed{1}$
(1x1) + (2x2) 1 + 4 = $\boxed{5}$
(1x1) + (2x2) + (3x3) 5 + 9 = $\boxed{14}$
(1x1) + (2x2) + (3x3) + (4x4) 14 + 16 = $\boxed{30}$

7.13.01.02A solve problems by simplifying the data by finding a pattern.
e.g.,

There were 2,386 votes cast for the two candidates for school president. The winner received 894 more votes than the loser. How many votes did the winner receive?

First Step: Substitute lesser numbers for the numbers given.

Second Step: Determine a method of solution which can then be used to solve the original problem.

7.13.01

VOCABULARY

TERMS	SYMBOLS
pattern model chart	

GETTING STARTED

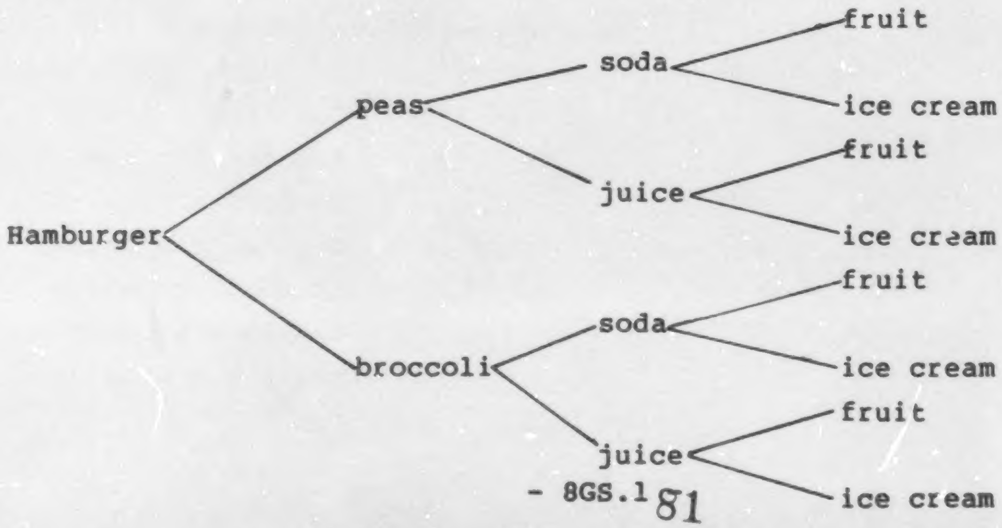
Write 3,6,9,--,15,--,--,24 on the chalkboard. Say, "What numbers belong in each blank?" Elicit 12,18,21. Say, "How did you know what numbers to fill in?" Elicit that a pattern exists where 3 is added to each number. Repeat this procedure using 1,2,4,7,--,16,--,29,-- and 25,24,--,11,--,-16,--.

Write this menu on the chalkboard.

Main Dish	Vegetable	Drink	Dessert
Hamburger	Peas	Soda	Fruit
Fried Chicken	Broccoli	Juice	Ice Cream

Say "The Big Apple Luncheonette offers these choices for dinner. How many times could you eat at the Big Apple without eating the same exact dinner twice?" Divide the students into groups of five and give them five minutes to determine an answer. Ask the students how they arrived at their answers. Draw a chart on the chalkboard.

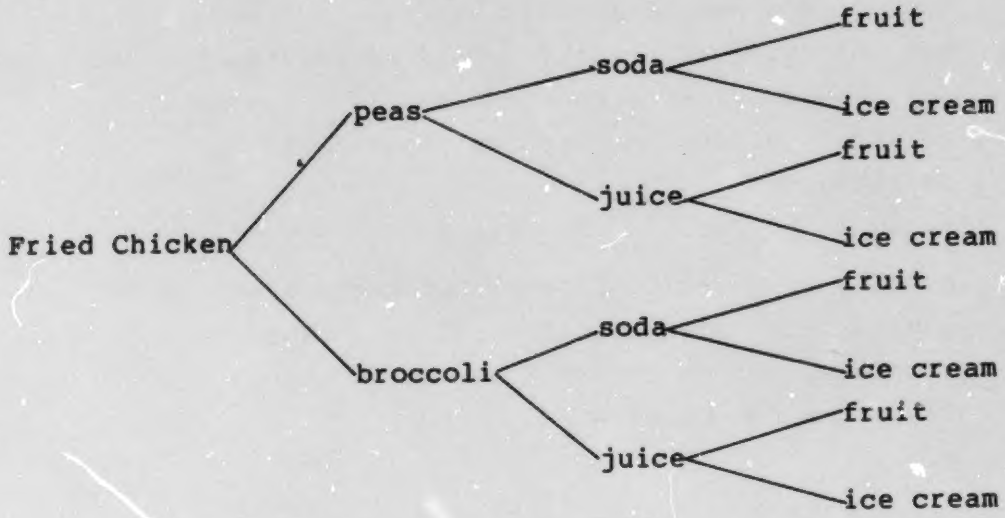
e.g.,



GETTING STARTED cont'd

Repeat counting the number of meals for fried chicken.

e.g.,



7.13.01

7.13.01.01 **use models/charts to simplify a problem**

STRATEGY 1: Present a problem on the chalkboard. If you add the digits in a number between 0 and 1000, how many of them have a sum of 10? (235 = 2 + 3 + 5 = 10)
Ask the students how they would solve this problem. Say "You can make a chart to see if a pattern exists." Write the chart headings on the chalkboard.

e.g., 1 - digit   2 - digit   3 - digit

Ask the students to list all of the numbers whose digits, when added, will equal ten. When this is complete, add to find out how many numbers are listed on the chart.

e.g.,

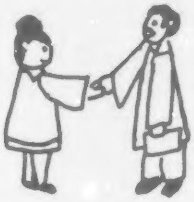
1 - digit	2 - digit	3 - digit
	19 91 55	235
	28 82	631
	46 64	244...
	37 73	
	9	3

STRATEGY 2: Write the following problem on the chalkboard.

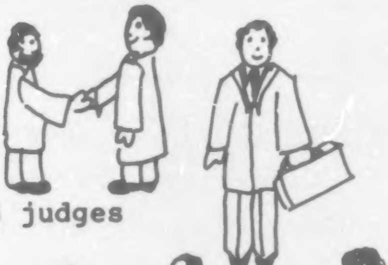
> "Whenever the nine Supreme Court Justices get ready to enter the courtroom, they shake each others hands. How many handshakes take place?"

Ask the students how they would determine the answer. Elicit several methods. Say that we are going to draw a model to help us determine the answer. Write and draw the following on the chalkboard.

STRATEGY 2: e.g.,
cont'd



2 judges = 1 handshake



3 judges = 3 handshakes

4 judges = 6 handshakes

Continue diagraming until nine judges are drawn.

7.13.01.02 **solve problems by simplifying the data by finding a pattern**

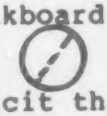
STRATEGY 1: Present the following problem.

A pie can be cut into 7 unequal
pieces using 3 straight cuts.
How many unequal pieces can be made
with 8 straight cuts?

Write the following on the chalkboard.

e.g.,

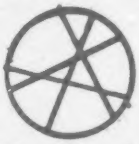
Number of Cuts	1	2	3	4	5	6	7	8	9	10
Number of Pieces										

Draw eight circles on the chalkboard. Ask a student to come to the chalkboard and make one cut on the first circle, e.g.,  . Say, "How many pieces are there?" Elicit that there are two pieces and ask the student to fill in the chart.

STRATEGY 1: cont'd

Repeat this procedure for each circle, reminding the students that each cut should cross the maximum number of lines.

e.g.,



Fill in the chart for five cuts.

e.g.,

Number of Cuts	1	2	3	4	5
Number of Pies	2	4	7	11	16

Say, "Without going any further, can we find a way to solve the problem?" Elicit that we need to find a pattern. Ask the student to study the chart. Elicit the pattern.

e.g.,

Number of Cuts	1	2	3	4	5
Number of Pieces	2	4	7	11	16

Say, "Using the pattern, how many pieces will there be if we make 6 cuts?" Elicit 16 + 6 = 22 pieces. Repeat this procedure for 7 cuts, 8 cuts. Check each answer with a diagram. Extend the problem by asking how many pieces will be made if 9 cuts are made.

STRATEGY 2:

Write the following on the chalkboard.

e.g.,

> The hallway at the video arcade has a pattern of 4 white tiles and 1 red tile. If 1000 tiles are needed, how many should be white and how many red?

Ask the students to construct a chart to solve this problem.

e.g.,

5 tiles = R W W W W = 1 red 4 white
10 tiles = R W W W W R W W W W = 2 red 8 white

7.13.01

STRATEGY 1: con't

Ask what happened between 5 and 10. Elicit that the number of each color is doubled. Say, "What will happen if we need 50 tiles?" Elicit 50 ÷ 5 = 10 and 10 x 1 red tile = 10 red tiles, 10 x 4 white tile = 40 white tiles. Write this in the chart. Say, "How can we check to see if we are correct?" Elicit 10 + 40 = 50 tiles. Using this procedure, elicit the number of each color tile for 100.

e.g.,

100 ÷ 5 = 20 and 20 x 1 red tile = 20 red tiles,
20 x 4 white tiles = 80 white tiles.

STRATEGY 2:

Say, "Look at the chart. Using the pattern in the chart, how can we generalize a solution for 1000 tiles?"

e.g.,

5 tiles	1 red	4 white
10 tiles	2 red	8 white
100 tiles	20 red	80 white

Elicit 200 red, 800 white. Say, "We have solved this problem by looking for a simpler but equivalent problem, solving it, and finding a pattern to help us solve the larger problem." Repeat this procedure using other problems that can be solved by finding a simpler and equivalent problem, solving it, and generalizing the solution.

P.O. 7.13.01.01

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Problem: How many squares are there in an 8 x 8 checkerboard?

Fill in the table. Draw actual boards.
Use the information to generalize a solution.

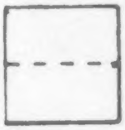
Size of the board.	Number of Squares Involving					Total number of Squares
	1 square	4 squares	9 squares	16squares	25squares	
1 x 1	1					1
2 x 2	4	1				5
3 x 3						
4 x 4						
5 x 5						

P.O. 7.13.01.02

NAME:________________________________ **CLASS:**________ **DATE:**_______

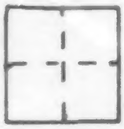
Solve the problem by making a chart or table and generalize your answer.

Problem: How many boxes will appear on a sheet of paper if you fold it in half 9 times?



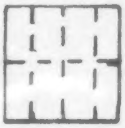
1 time =

2 boxes



2 times =

4 boxes



3 times

8 boxes

Number of Folds	Number of Boxes
1	2 boxes

*Remember to look for a pattern!

7.04.01 MULTIPLICATION OF LARGE NUMBERS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.04.01.01A compute the product of a 1-digit, 2-digit and a 3-digit multiplier;

e.g.,

$$\begin{array}{r} 3102 \\ \times \quad 3 \\ \hline 9,306 \end{array} \qquad \begin{array}{r} 216 \\ \times \quad 8 \\ \hline 1,728 \end{array} \qquad \begin{array}{r} 3567 \\ \times \quad 7 \\ \hline 24,969 \end{array}$$

$$\begin{array}{r} 876 \\ \times\ 40 \\ \hline 35,040 \end{array} \qquad \begin{array}{r} 504 \\ \times\ 52 \\ \hline 26,208 \end{array} \qquad \begin{array}{r} 322 \\ \times 136 \\ \hline 43,792 \end{array}$$

7.04.01.02A compute the product of a whole number multiplied by powers of ten;

e.g.,

$$\begin{array}{r} 34 \\ \times 10 \\ \hline 340 \end{array} \qquad \begin{array}{r} 367 \\ \times 100 \\ \hline 36,700 \end{array} \qquad \begin{array}{r} 5424 \\ \times\ 100 \\ \hline 542,400 \end{array} \qquad \begin{array}{r} 2437 \\ \times 1000 \\ \hline 2,437,000 \end{array}$$

7.04.01.03A apply the properties of multiplication: commutativity, associativity, distributivity (multiplication over addition), multiplicative property of zero, and the identity element;

e.g.,

$4 \times 6 = 6 \times 4$

$(3 \times 4) \times 7 = 3 \times (4 \times 7)$

$10 \times 6 = (9 \times 6) + (1 \times 6)$

$6 \times 0 = 0$

$6 \times 1 = 6$

7.04.01.04A solve verbal problems related to skills learned in this module.

e.g.,

Each box contains 18 pieces of chalk. If there are 15 boxes, how many pieces of chalk are there in all?

$$\begin{array}{r} 18 \\ \underline{\times 15} \\ 270 \end{array}$$ pieces of chalk

7.04.01

VOCABULARY

TERMS **SYMBOLS**

commutative
associative
distributive
identity element

GETTING STARTED

Write the following problems on the chalkboard.

1. 7+7+7+7+7 = □ x 7   2. □ + □ + □ = 3 x 9
3. 5 x 4 = □   4. 3 x 8 = □   5. 6 x 9 = □
6. 9 x 6 = □   7. 7 x 0 = □   8. 2 x 1 = □

Say that multiplication is a short form of addition of the same addends. Ask the students for the meaning of a product (result of multiplication) and factors (quantities multiplied to form a product). Elicit, using examples (5) and (6), that when two (2) factors are interchanged, the product is not affected. Using (7), elicit that the product of any number and 0 is 0. Using example (8), say that the product of a number and 1 is the number.

Prepare a set of cards with one 1-, 2- or 3-digit number on each. Divide the class into two teams, A and B. Ask one student from each team to come to the chalkboard. Hold up two (2) cards.

e.g.,

16   32

Ask the two students to multiply the two numbers. Whoever writes the correct product, first, receives a point for his/her team.

Repeat this procedure until one team reaches 10 points.

7.04.01

7.04.01.01 **compute the product of a 1-digit multiplier, 2-digit multiplier, and a 3-digit multiplier**

STRATEGY 1: Say, "A long distance runner ran 32 miles on each of 3 consecutive days. What was the total distance covered in the 3 days?"

Say that we need to find the sum of 32 + 32 + 32 or equivalently, 32 x 3. Ask students to set up the problem vertically, e.g., $\begin{array}{r} 32 \\ \underline{\times 3} \end{array}$. Say, "32 is called the multiplicand and 3 is called the multiplier." Say that in the vertical set-up, the ones digits are aligned. Elicit from the students that multiplication is always done from right to left beginning with the ones digit in the multiplicand.

Say the steps in multiplication, e.g., 3 x 2 = 6, so write $\begin{array}{r} 32 \\ \underline{\times 3} \\ 96 \end{array}$; then 3 x 3 = 9, so write $\begin{array}{r} 32 \\ \underline{\times 3} \\ 96 \end{array}$.
Say, "Hence the runner covered 96 miles."

Ask the students to multiply 632 by 4. Elicit that the problem is first set up as $\begin{array}{r} 632 \\ \underline{\times\ 4} \end{array}$ and then multiplied from right to left. Elicit the steps in multiplication of the problem.
e.g.,

Step 1: 4 x 2 = 8, write $\begin{array}{r} 632 \\ \underline{\times 4} \\ 28 \end{array}$ (Note: Reinforce that the 1 is carried to the next digit since we cannot write 12 in the tens place; 12 tens = 1 hundred + 2 tens, so the 1 hundred is "carried" to the hundreds place).

Step 2: 4 x 6 = 24, so we have 24 hundreds plus the 1 that was carried equal 25, and write $\begin{array}{r} 632 \\ \underline{\times 4} \\ 2528 \end{array}$.

STRATEGY 1:
cont'd

Ask the students to multiply

$$\begin{array}{r} 423 \\ \underline{\times 25} \end{array}$$

Elicit that the multiplier has 2-digits, and since 25 = 2 tens + 5 ones, we need to multiply 423 by 5 ones and then 2 tens. Say to the students that we will get two partial products as a result and we must add those partial products to get the final product.

Elicit the multiplication by 5 first to get the first partial product.

e.g.,

$$\begin{array}{rl} 423 & \\ \underline{\times 25} & \\ 2115 & \text{(1st partial product)} \end{array}$$

Elicit the multiplication by 2 to get the second partial product. Say that the second partial product must be indented one place to the left so that its right hand digit is under the tens place since we are multiplying by 2 tens and the product will thus be in tens.

e.g.,

$$\begin{array}{rl} 423 & \\ \underline{\times 25} & \\ 2115 & \text{(1st partial product)} \\ \underline{846} & \text{(2nd partial product)} \end{array}$$

Say that we now add the two partial products to get the final product.

e.g.,

$$\begin{array}{rl} 423 & \\ \underline{\times 25} & \\ 2115 & \\ \underline{846} & \\ 10575 & \text{(final product)} \end{array}$$

Repeat this procedure with other 2-digit multipliers.

Adapt the method used in this strategy for 3-digit multipliers. Say that we multiply with every digit in the multiplier, and that we place the right hand digit of each partial product directly under the digit that is doing the multiplying.

STRATEGY 2: Say to the students, "A dealer bought three (3) walkman-type radios for $31.00 each. What was the cost of all three (3)?" Elicit that the solution to this problem requires us to find the sum of 31 + 31 + 31 + 31 or equivalently $\begin{array}{r}31\\ \underline{\text{x}4}\end{array}$. Elicit that since 31 = 3 tens + 1 one, we must multiply 4 x 1 one (4 ones) and 4 x 3 tens (12 tens). (Note: Say to the students that multiplication is from right to left.) Hence, $\begin{array}{r}31\\ \underline{\text{x}4}\\ 124\end{array}$.

Ask students to multiply 432 by 6. Elicit that 432 = 4 hundreds + 3 tens + 2 ones. Elicit the multiplication steps from right to left.

e.g.,

6 x 2 ones = 6 x 2 = 12
6 x 3 tens = 6 x 30 = 180
6 x 4 hundreds = 6 x 400 = $\underline{2400}$
6 x 432 = 2592

Say, "We think through the multiplication above but we write $\begin{array}{r}432\\ \underline{\text{x}4}\\ 2592\end{array}$."

Repeat this procedure with other multiplication examples. Ask the students to multiply $\begin{array}{r}423\\ \underline{\text{x}25}\end{array}$. Elicit that since 25 = 2 tens and 5 ones, we multiply 423 x 5 ones, and then 423 x 2 tens to get the two partial products which we then add to get the final product.

Elicit 423 x 5 ones = 2115 and 423 x 2 tens = 8460. Say, "Since we are multiplying by 2 tens, we place the 6 in the tens place, and a 0 is used as a place holder in the ones place. Add 2115 + 8460 = 10575."

e.g.,

$$\begin{array}{rl} 423 & \\ \underline{\text{x}25} & \\ 2115 & \text{(1st partial product)} \\ \underline{+8460} & \text{(2nd partial product)} \\ 10575 & \text{(final product)} \end{array}$$

STRATEGY 2: cont'd Repeat this procedure using other multiplication examples. Adapt the method used in this strategy for 3-digit multipliers.

7.04.01.02 **compute the product of a whole number multiplied by powers of ten**

STRATEGY 1: Say "In a charity fund raiser, cyclists were given a $2.00 credit for charity for each mile they travel by bike. Three (3) cyclists traveled 10 miles, 100 miles, and 1000 miles respectively. How much did each earn for charity?"

Elicit that we need to multiply $\begin{array}{r}10\\ \text{x}2\\ \hline\end{array}$, $\begin{array}{r}100\\ \text{x}2\\ \hline\end{array}$, and $\begin{array}{r}1000\\ \text{x}2\\ \hline\end{array}$ respectively, and elicit the results of 20, 200, and 2000 respectively.
Repeat this procedure using similar examples.
Elicit that to find the product of a whole number and 10, add a 0 to the whole number; to find the product of a whole number and 100, add 2 zeros. Adding 3 zeros forms the product of a whole number and 1000. Say that we add the number of zeros that are found in the particular power of ten that we are using as a factor. Review the powers of ten, e.g., add 5 zeros when multiplying by 100,000.

Repeat this procedure with other multiplication problems with varied powers of 10.

STRATEGY 2: Say, "Three (3) different machines in a factory can produce 10, 100, and 1000 items per hour respectively. In three hours, how many items can each machine produce?"

STRATEGY 2: cont'd

Elicit that we need to multiply $\begin{array}{r}10\\ \underline{\times 3}\end{array}$, $\begin{array}{r}100\\ \underline{\times 3}\end{array}$, and $\begin{array}{r}1000\\ \underline{\times 3}\end{array}$ respectively. Say that 10 x 3 means 3 tens or 30, 100 x 3 means 3 hundreds or 300, and 1000 x 3 means 3 thousands or 3000.

Repeat this procedure using other examples where one factor is a power of ten.

7.04.01.03 **apply the properties of multiplication: commutativity, associativity, distributivity (multiplication over addition), multiplicative property of zero, and the identity element**

STRATEGY 1: Draw the following on the chalkboard.

e.g.,

XXXXX
XXXXX

Say that if the order of the factors is changed, the product is not changed. Say, "The property of multiplication illustrated is called the commutative property."

Write examples on the chalkboard.

e.g.,

6 x 9 = ☐ x 6, ☐ x 8 = 8 x 9, 1 x 7 = 7 x ☐

Ask the students to find the missing number to make each statement true.

Ask the students if it is possible to group (or associate) factors in different ways which would change the product. Ask the students to find the following products of factors regrouped by use of parentheses. Explain that the step is to multiply the factors within a parenthesis.

Elicit that (4 x 5) x 6 = 20 x 6 = 120 and
4 x (5 x 6) = 4 x 30 = 120.

STRATEGY 1: cont'd

Elicit that you can group factors in any order without affecting the product. Say, "This is the associative property of multiplication."

Ask the students to find the missing number to make the following statements true.

$(4 \times \square) \times 9 = 4 \times (5 \times 9)$

$8 \times (6 \times 2) = (8 \times 6) \times \square$

Write on the chalkboard, $(6 \times 9) \times 5 = (6 \times 5) \times 9$. Say, "Which is the easier example? Why?" Elicit that $(6 \times 5) \times 9$ is the easier of the 2 multiplications because multiplication by five is easy, and that by using the commutative and associative properties of multiplication we can do multiplications in a quicker and easier manner. Repeat this procedure using other similar examples.

Ask the students to find the value of $2 \times (3 + 4)$. Elicit that $2 \times (3 + 4) = 2 \times (7) = 14$.

Draw an illustration on the chalkboard.

e.g., xxxxxxx
xxxxxxx

Say that it represents $2 \times (7)$ or equivalently $2 \times (3 + 4)$. Say that if the diagram is partitioned into 2 sets, then we have illustrated $2 \times 3 + 2 \times 4$, which is equivalent to $2 \times (3 + 4)$.

e.g., xxx xxxx
xxx xxxx

Say, "We have an alternate way of finding the value of $2 \times (3 \times 4)$ called the distributive property of multiplication."

Ask the students to find the value of $6 \times (9 + 20)$ and similar problems to reinforce using the distributive property.

STRATEGY 1: cont'd

Elicit that multiplication can be made simpler by using the distributive property.
e.g.,
6 x 87 = 6 x (80 + 7) = 6 x 80 + 6 x 7 = 480 +42 = 522
Ask the students to find the products of several examples.
e.g.,
8 x 0, 8 x 1, 24 x 0, 24 x 1, 407 x 0, 407 x 1.

Elicit that the product of any number and 0 is 0. Say that the multiplication property of 0 is illustrated here.

Say that the product of any number and 1 is that identical number and call 1 the identity element of multiplication.

STRATEGY 2:

Ask the students to find the products of 8 x 3 and 3 x 8. Elicit from this and similar examples that changing the order in which numbers are multiplied does not change the product. Say, "This illustrates the commutative property of multiplication."

Ask the students to multiply 5 x 3 x 2. Elicit the steps to multiply.
e.g.,

(1) 5 x 3 = 15
(2) 15 x 2 = 30
or
(1) 3 x 2 = 6
(2) 15 x 2 = 30

Say, "(5 x 3) x 2 = 5 x (3 x 2) = 30. Therefore, grouping (associating) factors differently does not affect the product. This is called the associative property of multiplication."

STRATEGY 2: cont'd

Say that the commutative property can be used to make multiplications simpler and use these examples.

e.g.,

8 x □ = 0, 8 x □ = 8, □ x 24 = 0, □ x 24 = 24.

Say that the product of any number and zero is zero, and that the product of any number and 1 is that identical number. Explain to the students that 8 x □ = 0, □ x 24 = 0 illustrates the multiplication property of 0 and we call 1 the identity element of multiplication. Repeat this procedure using other examples which show the rules of multiplication.

7.04.01.04 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following problem on the chalkboard.

> A certain plant takes 32 weeks to mature. How many days will it take to mature?

Elicit that one week contains 7 days, so 32 x 7 = 224 days.

Write on the chalkboard the following problem.

e.g.,

> In a New York high school there are 45 classes with 32 students per class. What is the total number of students in the school?

Elicit that 45 x 32 = 1440 students.

Write another problem on the chalkboard.

STRATEGY 1: cont'd

e.g.,

A dealer sold 100 cars at $5625.00 each. Find his total amount of sales.

Elicit that 100 x $5625.00 = $562,500.00

Repeat this procedure using other verbal problems.

STRATEGY 2: Write a problem on the chalkboard.
e.g.,

A new power plant will take 9 years to build. How many days will it take to build?

Elicit that 1 year contains 365 days, so
9 x 365 = 3285 days.
Write another problem on the chalkboard.
e.g.,

A printing press prints 28 books each day. If each book contains 276 pages, how many pages are printed each day?

Elicit 28 x 276 =7728 pages.

Write the following problem on the chalkboard.
e.g.,

A certain community consumes 8564 gallons of gasoline in a day. How many gallons are consumed in 1000 days?"

Elicit that 8564 x 1000 = 8,564,000 gallons.

Repeat this procedure using other verbal problems.

P.O. 7.04.01.01

NAME:__________________________________CLASS:________DATE:________

Find the product.

1. 24 x2	2. 63 x3	3. 514 x 6	4. 2807 x 8
5. 53 x32	6. 614 x 40	7. 3065 x 75	8. 47328 x 91
9. 516 x312	10. 708 x607	11. 3879 x 219	12. 60544 x 495

P.O. 7.04.01.02

NAME: ____________________ CLASS: __________ DATE: ________

Replace the □ to make true statement.

1. 35 x □ = 3500

2. □ x 273 = 2730

3. 81 x □ = 81000

4. □ x 40 = 4000

5. □ x 4725 = 47250000

6. 7 x □ = 70.

Multiply.

7. 100 x 56 = □

8. 10 x 728 = □

9. 96 x 1000 = □

10. 2314 x 100 = □

11. 10000 x 8040 = □

*12. 9 x 4 x 10 = □

*13. 100 x 6 x 12 = □

P.O. 7.04.01.03

NAME: ____________________ CLASS: __________ DATE: ________

1. Name the multiplication property that is illustrated by each mathematical sentance.

1. 8 x (9x7) = (8x9) x 7.

2. 6 x 1 = 6

3. 4 x (7 + 2) = 4 x 7 + 4 x 2

4. 0 x 25 = 0

5. 4 x 6 = 6 x 4

2. In each problem: (a) Replace the ☐ to make the mathematical sentence true and (b) name the multiplication property that each completed sentence illustrates.

6. ☐ x 8 = 8

7. 4 x (9+6) = 4 x ☐ +4x6

8. 3 x (5 x 4) = (☐ x 5) x 4

9. 7 x ☐ = 2 x 7

10. 9 x ☐ = 0

11. ☐ x (8 + 1) = 5x8 + 5x1

3. Find the following products in 3 ways.

12 3 x 5 x 7

13. 2 x 8 x 5

P.O. 7.04.01.04

NAME:______________________________ CLASS:________ DATE:_______

Solve each problem.

1. A flock of 72 birds flew 26 miles one day. What was the total number of miles flown by the flock in 15 days?

2. A dealer sold 37 T.V. sets at $429.00 each. Find the total amount of the sales.

3. Light travels at 186,200 miles in one second. How far does it travel in 345 seconds?

4. A truck holds 125 boxes, each of which weighs 107 pounds. Find the total weight of the boxes in the truck.

7.05.01 EXPRESS A REMAINDER AS A FRACTION

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.05.01.01A express a remainder in a division example as a fraction;

e.g.,

$$\begin{array}{r} 3\frac{3}{7} \\ 7\overline{)24} \end{array}$$

7.05.01.02A solve verbal problems related to skills learned in this module.

e.g.,

Susan invited seven friends to her birthday party. Her mother baked 24 cupcakes for the party. How many cupcakes should each person at the party get?

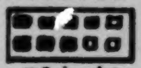
$$\begin{array}{r} 3\frac{3}{7} \text{ cupcakes} \\ 7\overline{)24} \end{array}$$

VOCABULARY

TERMS

remainder
fraction
numerator
denominator

GETTING STARTED

Place a simple fraction on the chalkboard, e.g., $\frac{3}{10}$. Ask the students to identify the parts of the fraction, e.g., numerator, denominator, fraction bar. Elicit that the denominator stands for the number of parts in a whole, and the numerator stands for the number of parts involved in the problem. Draw a picture of an ice cube tray, e.g.,  . Say, "How many ice cubes does the ice cube tray hold?" Elicit that it takes ten cubes to make one whole tray. Say, "10 stands for the denominator. How many ice cubes are missing from the tray?" Elicit that seven ice cubes are missing. Say, "The number of cubes that are involved in the problem is 7 and so the numerator is 7." Say, "What fraction is represented?" Elicit that the fraction of missing ice cubes is $\frac{7}{10}$. Repeat this procedure using other examples.

Place a simple division problem on the chalkboard, e.g., $5\overline{)22}$. Ask the students if this problem can be written any other way. Elicit that a division problem can be written in three different ways.
e.g., $5\overline{)22}$ or $22 \div 5$ or $\frac{22}{5}$
Place several different division problems on the chalkboard and ask the students to write the different ways of expressing each of the division problems.
e.g.,

fraction form (—)	$\overline{)}$ Form	÷ Form
$\frac{22}{5}$	☐	☐
☐	$3\overline{)47}$	☐
☐	☐	16÷7

7.05.01

7.05.01.01 **express a remainder in a division example as a fraction**

STRATEGY 1: Say, "Fractions are a form of division and every fraction can be put into division form, e.g., $\frac{5}{2} = 2\overline{)5}$, $\frac{3}{2} = 2\overline{)3}$."

Explain that when we have a remainder in a division problem, it's the same as having a fractional part left over in a mixed number. Demonstrate that we can express the remainder as a fraction by placing the remainder over the divisor, thereby, making it a fraction.

e.g.,

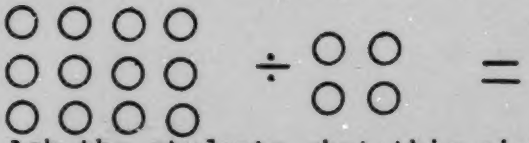
$$\begin{array}{r} 6\text{r}1 \\ 3\overline{)19} \\ \underline{18} \\ 1 \end{array} \quad = \quad 6\frac{1}{3}$$

Place several examples on the chalkboard to repeat the procedure.

e.g.,

$$\begin{array}{r} 6\frac{1}{4} \\ 4\overline{)25} \\ \underline{24} \\ 1 \end{array} \qquad \begin{array}{r} 3\text{r}7 \\ 8\overline{)31} \\ \underline{24} \\ 7 \end{array} = 3\frac{7}{8}$$

STRATEGY 2: Place the following picture on the chalkboard.



Ask the students what this picture represents. Elicit that this is a picture of a division problem and that other names for this problem could be $\frac{15}{4}$ or 15 ÷ 4. Say, "All these examples ask the same question. What is the question?" Elicit how many groups of 4 can be taken away from 15. Say, "We all know that three groups of 4 can be taken away from 15, but how can we express the amount that is left

STRATEGY 2: cont'd

over?" Write down the different answers, $\frac{15}{4} = 3\frac{3}{4}$, $\overset{3r3}{4\overline{)15}}$. Say, "Writing a quotient with a remainder is only one way of expressing the answer. The remainder represents the part that is left over after we take out all the 4s, and therefore, the remainder is that part of 4 that is left over. How else can we express a remainder?"

Elicit that a remainder can be expressed as a fraction because a fraction also represents the number of parts involved (numerator) over the number of parts in one whole thing (denominator).

Say that $\overset{5r1}{4\overline{)21}}$ means the same thing as $\overset{5\frac{1}{4}}{4\overline{)21}}$, because $\frac{1}{4}$ means that there is one part left out of 4. Repeat this procedure using $3\overline{)20}$, $7\overline{)40}$ and $10\overline{)101}$.

7.05.01.02 **solve verbal problems related to skills learned in the module**

STRATEGY 1: Say, "What are some important things to look for in a verbal problem?" Elicit that it is important: (1) to read the problem carefully, (2) to find out what is being asked, (3) to determine what information is being given and (4) to determine how the problem can be solved using the given information. Say, "What key words tell us the operation to use?" Elicit that in a division problem the key words are how many are in each group, how many groups can be made, how are they shared and how many did each get. Write a verbal problem involving division on the chalkboard.

STRATEGY 1: cont'd

e.g.,

> "Susan brought 26 cupcakes to her club meeting. If there were 7 people at the meeting, how many cupcakes would each person get?"

Elicit that this problem contains one of the questions that shows this is a division problem. Ask a student to come to the chalkboard to estimate his/her answer and then solve the division problem, e.g., $7\overline{)26} = 3\frac{5}{7}$ cupcakes. Make sure that the word cupcakes is part of the answer. Repeat this procedure using other verbal problems involving division with a remainder.

STRATEGY 2: Write a division problem on the chalkboard, e.g., $15\overline{)201}$. Ask the students to use this division example to construct a verbal problem and solve it.

e.g.,

> There are 201 children in the 5th grade. If each class has 15 children, how many classes will there be?

Ask several students to come to the chalkboard and place their verbal problems on the chalkboard. From these sentences, pick out the key expressions that tell us that these are division problems and underline them. Some of these expressions are should be made, how are they shared, how many did each get.

Ask several students to come to the chalkboard and write an example to solve his/her problem, estimate the answer and then actually solve the problem, labeling his/her answer.

Repeat this procedure using other students' examples.

P.O. 7.05.01.01

NAME:____________________ CLASS:________ DATE:_______

Express each quotient as a mixed number.

Example: 3 r 3 = $3\frac{3}{7}$ 7)24	7 r 1 = 8)57	7 r 1 = 6)43
3 r 3 = 8)27	11 r 6 = 7)83	19 r 4 = 5)99
7 r 3 = 4)31	4 r 3 = 4)19	6 r 2 = 9)56
19 r 2 = 3)59	13 r 3 = 7)94	29 r 1 = 2)59

P.O. 7.05.01.02

NAME: ______________________________ CLASS: __________ DATE: ________

Solve each problem. Express answers as mixed numbers.

1. Jack took a trip on a jet. He flew 3,850 miles in 6 hours. What was the average rate of the jet per hour?

2. In 2 weeks (14 days), Mr. Smith sold 1,302 newspapers. How many did he sell each day?

3. In Lake Canoe day camp there are 483 campers. If there are to be 24 campers in each group, how many groups can be made?

4. Each crate of oranges holds 144 oranges. If there are 16,000 oranges. How many crates will be needed?

5. On a camping trip the family plans to travel 863 miles in 4 days. How many miles should they plan to drive each day?

6. The Spiros took out a car loan for $5,860. If the loan is to to be repaid in 36 monthly installment. What is the amount of each payment?

7. If each can of insect spray can kill about 250 insects, how many cans would be necessary to kill 7,865 insects?

8. The girls bought 6 pounds of hamburger for $8.34. What was the price per pound?

7.01.02 DIVISIBILITY RULES: 2,5,10

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.02.01A identify and use the divisibility rule for 2;

Note: All numbers with an even units digit (zero is even) are divisible by two.

e.g., 644

7.01.02.02A identify and use the divisibility rule for 5;

Note: If the units digit of a number is 0 or 5, it is divisible by 5.

e.g., 640 , 635

7.01.02.03A identify and use the divisibility rule for 10.

Note: If a number has its units digit 0, it is divisible by 10.

e.g., 6,480

7.01.02

VOCABULARY

TERMS

divisible
digit
units digit
factor
odd
even

GETTING STARTED

Write the following on the chalkboard.

46 ÷ 2, 78 ÷ 5, 850 ÷ 10, 496 ÷ 9

Ask, "In which example is the remainder zero?" Elicit that the remainder is zero in the first and third divisions. Say to the students that one number is divisible by another number if the quotient is a whole number with a remainder of zero. 46 is divisible by 2 since 46 ÷ 2 = 23 with 0 remainder, and 78 is not divisible by 5 since 78 ÷ 5 = 15 with a remainder a 3. Repeat this procedure using other division examples.

Play a factor game. Divide the class into two teams, A and B and choose a scorekeeper. Write $\frac{\text{Number}}{\text{Factors}}$ on the chalkboard. The first member of team A comes to the chalkboard and writes a number where it says "Number." A limit to the size of the number may be made, e.g., 1 through 100, 1 through 200. The first member of team B comes to the chalkboard and writes as many of the factors for that number as he/she knows. The number of factors are added and that is the team's score for that round.

e.g.,	Number	36	Score = 9
	Factors	1, 2, 3, 4, 6, 9, 12, 18, 36	

Continue by alternating the team which chooses a number and the team which writes the factors until all students have had a turn. Determine the winner by asking the scorekeeper to add the totals.

7.01.02

7.01.02.01 identify and use the divisibility rule for 2

STRATEGY 1: Write 14, 18, 19 and 35 on the chalkboard. Ask the students if they can determine, without dividing, if any of these numbers is divisible by 2. Elicit that even numbers (numbers with unit digits of 0, 2, 4, 6, or 8) are divisible by 2. Say, "Which numbers are not divisible by 2?" Elicit that odd numbers (numbers with a unit digit of 1, 3, 5, 7, 9) are not divisible by 2. Write the numbers 246, 3,547, 183,958 and 10,009 on the chalkboard. Ask the students to determine which numbers are divisible by 2.

STRATEGY 2: Ask the students to name all of the factors for 30. Elicit and write on the chalkboard 1, 2, 3, 5, 6, 10, 15, 30. Ask the students to list the factors of 8 and 14 and write all the factors on the chalkboard. Say, "What are the common factors of 30, 8 and 14?" Elicit 1 and 2. Say to the students that 1 is a factor of all numbers and, therefore, 2 is the common factor of 30, 8 and 14.

Ask, "What do the numbers 30, 8 and 14 have in common?" Elicit that they are even numbers. Ask the students to define an even number. Say, "What is the rule pertaining to even numbers and the divisibility of a number by 2?" Elicit that all numbers which are divisible by 2 are even numbers and have a unit digit of 0, 2, 4, 6 or 8.

Repeat this procedure using other whole numbers. Determine if they are divisible by 2.

7.01.02

7.01.02.02 identify and use the divisibility rule for 5

STRATEGY 1: Divide the class into groups of four students. Say, "We are going to play a factor game and these are the rules.

1. Four numbers will be written on the chalkboard.
2. As soon as a student determines the largest common factor of all four numbers, he/she should stand.
3. The first person to stand in the group will state his/her answer.
4. Only one person in a group is allowed to stand."

Begin the game by writing 10, 55, 65, and 15 on the chalkboard. Continue using 20, 50, 35, 45, and 100, 90, 75, 25. Elicit the answer. Say, "These numbers all have 5 as their greatest common factor. What can we say about numbers which are divisible by 5?" Elicit that they all have a units digit of either 0 or 5.

STRATEGY 2: Ask the students to list the numbers which have 5 as a factor. Check by division. Elicit that each of the numbers which have 5 as a factor is divisible by 5.

Ask, "Can we determine without dividing, if a given number is divisible by 5." Elicit that from the above student examples, numbers which are divisible by 5 have a units digit of 0 or 5.

7.01.02.03 identify and use the divisibility rule for 10

STRATEGY 1: Say, "We are going to play a multiplication game." Divide the class into two teams. One (1) member of each team will go to the chalkboard. Say, "I am going to say two (2) numbers to each of you, e.g., 6 and 7. You are to add the numbers and multiply the sum by 10. Write the answer on the chalkboard. The first person to write the correct answer first gets a point. All correct answers remain on the chalkboard." Play the game until all of the students have had a chance. Determine which team has won. Ask the students to look at the numbers on the chalkboard and see what is common to all of them. Elicit that all are all divisible by ten and all have a 0 as the units digit.

STRATEGY 2: Ask the students to list numbers which have 10 as a factor. Ask the students to check the suggested numbers by division. (The quotient should be a whole number with no remainder.) Say, "Each of the numbers which have 10 as a factor is divisible by 10." Ask the students if we can determine, without dividing, if a given number is divisible by 10. Elicit from the above examples of numbers divisible by 10, that each has a units digit of 0.

Ask the students for examples of numbers which are not divisible by 10. Elicit that the units digit of such numbers is never 0 and, therefore, if the units digit of a number is 0, then the number is divisible by 10.

Write various numbers on the chalkboard (some of which are divisible by 10). Ask the students to state which numbers are divisible by 10 by using the divisibility rule they have just learned.

P.O. 7.01.02.01

NAME:______________________________ CLASS:________ DATE:________

(1) Circle the numbers that are divisible by 2:

57, 94, 409, 320, 786, 1053, 4090, 81732

(2) Circle the number pairs in which both numbers in the pair are divisible by 2.

(a) 84,50 (b) 76,47 (c) 0,138 (d) 42,241

(3) In the space below write all numbers which are divisible by 2 which fall between the numbers given:

(a) 19 and 27 (b) 109 and 130 (c) 4994 and 5010.

(4) Name a 3 - digit number that is: (a) divisible by 2

(b) not divisible by 2

(5) Complete the last digit of each number to make the number divisible by 2.

(a) 45__ (b) 3,77__ (c) 15,89__ (d) 50__

P.O. 7.01.02.02

NAME: ______________________ CLASS: ________ DATE: _______

(1) Circle the numbers that are divisible by 5.

52, 1745, 90, 40, 4080, 820, 0, 297305, 315

(2) Circle the number pairs in which both numbers in the pair are divisible by 5.

(a) 50, 195 (b) 103,85 (c) 415,3000 (d) 8270,65

(3) In the space below, write all numbers which fall between the given numbers and are divisible by 5.

(a) 38 and 72 (b) 107 and 145 (c) 6855 and 6910.

(4) Name a 4 - digit number which is: (a) divisible by 5

(b) not divisible by 5.

(5) Complete the last digit of each number to make the number divisible by 5.

(a) 8__ (b) 7,10__ (c) 14,52__ (d) 78__

P.O. 7.01.02.03

NAME: ______________________ CLASS: __________ DATE: ________

(1) Circle the numbers that are divisible by 10.

805, 4080, 60, 9001, 54070, 210004, 700

(2) Circle the number pairs in which both numbers in the pair are divisible by 10.

(a) 40,8005 (b) 210,0 (c) 19600,540 (d) 25,1000

(3) In the space below, write all the numbers which fall between the given numbers and are divisible by 10.

(a) 38 and 114 (b) 670 and 805 (c) 4320 and 4400

(4) Name a 5 - digit number which is: (a) divisible by 10

(b) not divisible by 10

(5) Complete the last digit of each number to make the number divisible by 10.

(a) 36_ (b) 84,07_ (c) 4_ (d) 9,00_

7.01.03 DIVISIBILITY RULES: 3,4,6,9

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.01.03.01A identify and use the divisibility rule for 3;

Note: Add the digits of the number. If the sum is divisible by 3, the number is divisible by 3.

e.g., 612

$6 + 1 + 2 = 9$

9 is divisible by 3 therefore, 612 is divisible by 3.

7.01.03.02A identify and use the divisibility rule for 4;

Note: Look at the last 2 digits of a number. If that number is divisible by 4, the whole number is divisible by 4.

e.g., 632 Since 32 is divisible by 4, 632 is also divisible by 4.

7.01.03.03A identify and use the divisibility rule for 6;

Note: If the number is divisible by 2 and 3, it is divisible by 6.

e.g., 426

It is even.

$4 + 2 + 6 = 12$

7.01.03.04A identify and use the divisibility rule for 9.

Note: If the digits of a number add up to a multiple of nine, it is divisible by nine.

e.g., 81

8 + 1 = 9

121

VOCABULARY

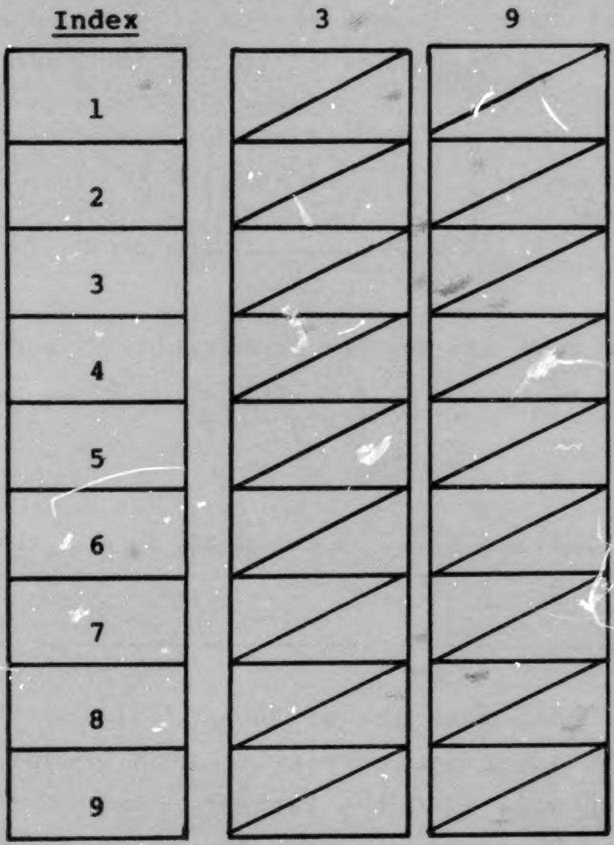
TERMS

divisible
digit
factor
multiple
common factor

GETTING STARTED

Prepare and distribute three (3) strips divided in this manner.

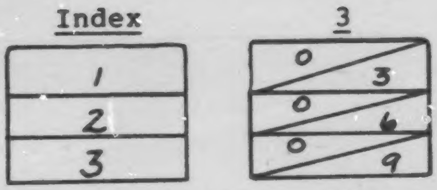
e.g.,



Ask the students to place the "Index" strip next to the "3" strips to fill in the multiplication table as follows.

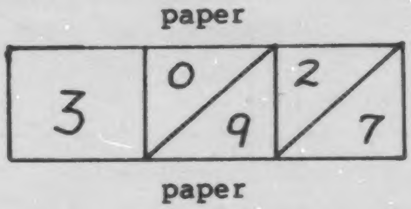
GETTING STARTED cont'd

e.g.,

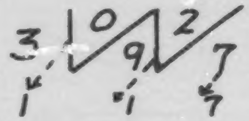


Then repeat for the "9" table. Say to the students that they can perform "magic multiplication" by using these 3 strips. Ask the students to line up the strips, one next to the other, with the Index strip on the left. Say, "We can find the product of 5 x 39 by using these strips. Place 2 sheets of paper across the strips so that only the 3 on the Index strip and the rows next to it show.

e.g.,



If we add the rows diagonally from right to left, we get



and 3 x 39 = 117."

Continue multiplying other numbers by 39 and then reverse the strips to multiply by 93.

Play a factor game with the students. Divide the class into groups of 4. Say, "I am going to write four (4) numbers on the chalkboard and each group will find the largest common factor of the four (4) numbers within 30 seconds. When I say 'Stop' each team will be asked for the largest common factor. If your group is correct the group will receive 1 point."

Start the game by writing 24, 36, 66 and 42 on the chalkboard.

Repeat, using other sets of four (4) numbers, varying in difficulty.

7.01.03

7.01.03.01 **to identify and use the divisibility rule for 3**

STRATEGY 1: Ask the students to tell which of the following numbers are divisible by 3. Write the numbers on the chalkboard, "57, 320, 163, 2541, 1874, 15807."

Elicit that 57, 2541, and 15807 are divisible by 3 while 320, 163, and 1874 are not. (Check by division by 3. Elicit that if a number is divisible by 3, the remainder is 0.)

Ask the students if some pattern exists in the numbers divisible by 3 so that we can use the pattern to develop a divisibility rule for 3, so that we do not have to divide. Allow students time for trial and error discovery of possible patterns. Follow through on their suggestions to see if the patterns work for all numbers divisible by 3.

If students cannot find a pattern, suggest to them that they add the digits of the numbers which are divisible by 3. Say, "What is special about the sum?" Elicit that the sum of the digits of a number which is divisible by 3 is also divisible by 3, e.g., 2541 = 2 + 5 + 4 + 1 = 12, and 12 ÷ 3 = 4. Repeat the test with other numbers which are divisible by 3.

STRATEGY 2: Adapt Strategy 1 used in P.O. 7.01.05.02 for divisibility by 3.

7.01.03.02 to identify and use the divisibility rule for 4

STRATEGY 1: Ask the students to name some numbers which are divisible by 4, and some which are not divisible by 4. Encourage the students to suggest large as well as small numbers. Check each number using division by 4. (If divisible, the remainder is 0.) Write the suggested numbers on the chalkboard under two (2) columns labeled divisible and not divisible.

Ask the students if a pattern exists in numbers divisible by 4 which does not exist for numbers not divisible by 4, so that a divisibility rule for 4 can be developed so not to have to actually divide. Allow students time for trial and error discovery of possible patterns. Follow through on their suggestions.

If the students cannot find a pattern, suggest to them that they examine the last two (2) digits of the numbers. Elicit that in numbers which are divisible by 4 the last two (2) digits of the number, if looked at as a separate 2-digit number, will also be divisible by 4. Write a set of numbers on the chalkboard some of which are divisible by 4, and some of which are not, and ask the students to identify which numbers use the divisibility rule just developed.
e.g., 178, 496, 1394, 8435, 625120, 421132

STRATEGY 2: Ask the students to state which of the following numbers are divisible by 4. Write the numbers on the chalkboard: 940, 153, 7916, 36912, 10846, 9818. Elicit that 940, 7916, and 36912 are divisible by 4, and 153, 10846, and 9818 are not

STRATEGY 2: cont'd

divisible by 4. Check for division by 4. Reinforce that if a number is divisible by 4, the remainder is 0.

Ask the students if some pattern exists in the numbers which are divisible by 4 which we can use to develop a divisibility rule for 4, so as not to have to divide. Allow students time for trial and error discovery of possible patterns. If students cannot find a pattern, suggest to them that they examine the last two (2) digits of the numbers. Elicit that in the numbers which are divisible by 4, the last two (2) digits are also divisible by 4.

Write a set of numbers on the chalkboard and ask the students to identify which numbers are divisible by 4 by using the divisibility rule just developed, e.g., 178, 496, 1394, 8435, 625120, 421132.

7.01.03.03 **to identify and use the divisibility rule for 6**

STRATEGY 1:

Ask the students to name some numbers which are divisible by 6. Encourage the students to use large as well as small numbers. Check each number using division by 6.

Say, "Does a pattern exist in these numbers?" Allow students time for trial and error discovery of possible patterns.

If students cannot find a pattern, ask the students to name all factors of 6 other than 1 and itself.

STRATEGY 1: cont'd

Elicit that the factors are 2 and 3. Say that since 6 = 2 x 3, a number divisible by 6 must be divisible by 2 and by 3.

Ask the students to state the divisibility rules for 2 and for 3. Elicit that both these rules must work for any number that is divisible by 6, namely, the number must be even (rule for 2), and the sum of the digits must be divisible by 3 (rule for 3). Write 636 on the chalkboard. Use both rules, e.g., 636 is an even number and 6 + 3 + 6 = 15 and 15 is divisible by 3.

Write another set of numbers on the chalkboard and ask the students to tell which are divisible by 6 using the divisibility rule just developed.

STRATEGY 2: Adapt Strategy 2 used in P.O. 7.01.05.01 for divisibility by 6.

7.01.03.04 to identify and use the divisibility rule for 9

STRATEGY 1: Write the numbers on the chalkboard: 426, 828, 4335, 81089, 32526, 9657. Ask the students to tell which numbers are divisible by 9.
Elicit that 828, 32526 and 9657 are divisible by 9, and 426, 4335, and 81,089 are not. (Check by division by 9.)

Ask the students to identify any patterns that exist in the numbers which are divisible by 9. Allow students time for trial and error discovery of possible patterns.

If students cannot find a pattern, ask them to add the digits of the numbers which are divisible by 9

STRATEGY 1: cont'd — and to compare their sums. Elicit that the sum of the digits of the numbers divisible by 9 is a multiple of 9, e.g., 81 = 8 + 1 = 9.

STRATEGY 2: Adapt Strategy 1 used in P.O. 7.01.05.02 for divisibility by 9.

P.O. 7.01.03.01

NAME:____ CLASS:_________ DATE:_______

1. Circle the numbers that are divisible by 3.

   54, 113, 528, 0, 1872, 427, 75193

2. Circle the number pairs in which both numbers in the pair are divisible by 3.

   (a) 81 and 429 (b) 600 and 5410 (c) 75 and 450

   (d) 255 and 183

3. In the space below, write all numbers which fall between the given numbers and are divisible by 3.

   (a) 45 and 70 (b) 96 and 130 (c) 5520 and 5562

   ________________ ______________ __________________

4. Name a 4-digit number which is:

   (a) Divisible by 3 (b) not divisible by 3

5. Complete the last digit of each number to make the number divisible by 3

   (a) 25__ (b) 9214__ (c) 7__ (d) 50__

P.O. 7.01.03.02

NAME: ____ CLASS: __________ DATE: ________

1. Circle the numbers that are divisible by 4.

842, 2560, 96, 50000, 7210, 325

2. Circle the number pairs in which both numbers in the pair are divisible by 4.

(a) 88 and 6240 (b) 0 and 356 (c) 5132 and 477

(d) 616 and 92

3. In the space below, write all numbers which fall between the given numbers, and are divisible by 4.

(a) 53 and 85 (b) 104 and 138 (c) 1980 and 2008

__________ ____________ _____________

4. Name a 5 digit number which is:

(a) Divisible by 4 (b) not divisible by 4

5. Complete the last digit of each number to make the number divisible by 4.

(a) 5_ (b) 26_ (c) 82,17_ (d) 3,20_

P.O. 7.01.03.03

NAME: ______________________ CLASS: ________ DATE: _______

1. Circle the numbers that are divisible by 6.

   89, 450, 0, 8176, 581745, 2844.

2. Circle the number pairs in which both numbers in the pair are divisible by 6.

   (a) 46 and 810 (b) 1236 and 300 (c) 114 and 2412 (d) 4800 and 241

3. In the space below, write all numbers which fall between the given numbers and are divisible by 6.

   (a) 35 and 80 (b) 246 and 310 (c) 3480 and 3540.

4. Name a 4-digit number which is:

   (a) divisible by 6. (b) not divisible by 6.

5. Complete the last digit of each number to make the number divisible by 6.

   (a) 8_ (b) 35_ (c) 4115_ (d) 640_

P.O. 7.01.03.04

NAME:______________________________________CLASS:__________DATE:________

Fill in the squares of all numbers that are divisible by nine.

MAZE KRAZE

24	43	78	284	614	312	418	719	223
18	29	56	123	810	774	522	639	261
36	82	114	516	441	357	462	831	801
126	37	231	129	333	285	489	516	324
45	28	625	148	END	329	331	626	135
315	63	234	423	144	72	351	711	54
80	178	132	29	184	906	809	489	726
240	64	93	103	821	251	127	421	357

7.09.01 ANGLE CLASSIFICATION

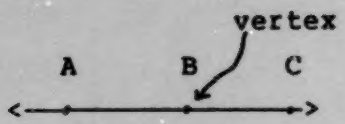
(2-3 lessons)

PERFORMANCE OBJECTIVES

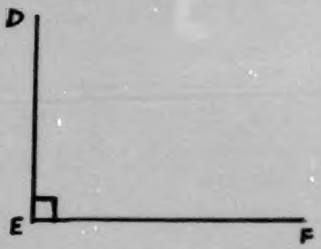
The student will be able to...

7.09.01.01A name and define the different types of angles;

e.g.,



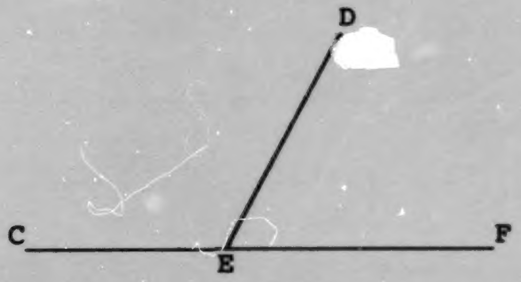
∠ ABC is a straight angle



∠ DEF is a right angle.

7.09.01.02A define and identify adjacent, supplementary and complementary angles.

e.g.,



∠ DEF and ∠ DEC are supplementary angles.

7.09.01

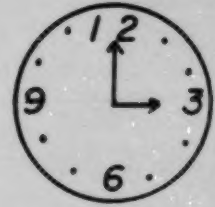
VOCABULARY

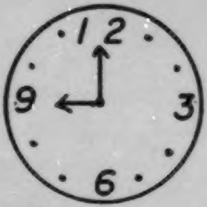
TERMS	SYMBOLS
vertex	
ray	—>
measure of angle	(m) ∡
adjacent	
complementary	
supplementary	

GETTING STARTED

Ask the students to look around the classroom at the windows, closets, chalkboard, lockers. Say, "What kind of angles are formed by these objects?" Elicit right angles are formed. Ask the students to observe the hands on a clock. Draw a model of a clock face on the chalkboard.

e.g.,

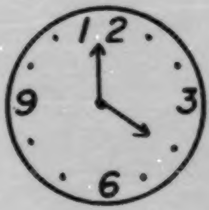


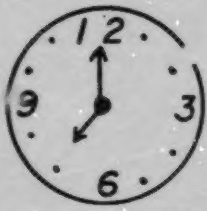


Draw the hands at different times of the day, e.g. 4:00.

Say, "What kind of angle is formed by the hands of the clock?" Elicit an obtuse angle is formed. Repeat this procedure with other times of the day.

e.g.,



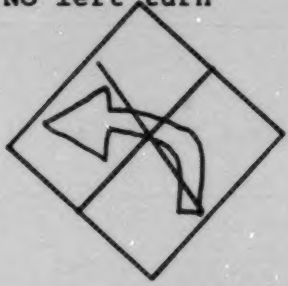


GETTING STARTED cont'd

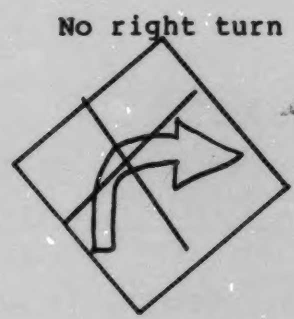
Say, "What kinds of hand signals are used by drivers?" Demonstrate hand signals, e.g., hand straight out or left turn, hand pointing up, bent at the elbow for right turn. Repeat for all road signs which use angles to signal drivers.

e.g.,

No left turn



No right turn

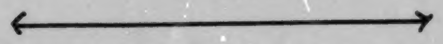


7.09.01

7.09.01.01 **name and define the different types of angles**

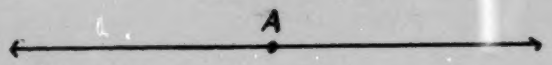
STRATEGY 1: Prepare and distribute a protractor and two rulers or pencils or sticks, to each student. Rotate one ruler until a straight line is formed with the other. Repeat this several times. Ask the students to experiment using their rulers or sticks. Ask the students what angle is formed by this rotation. Draw a diagram on the chalkboard.

e.g.,



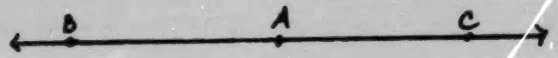
Elicit a straight angle. Ask a student where the vertex of this angle would be. Elicit that it would be at the center. Ask a student to locate and name the vertex in the diagram on the chalkboard.

e.g.,



Ask another student to name the sides of the angle.

e.g.,



Ask two students to name the angle.

Elicit ∢ BAC or ∢CAB.

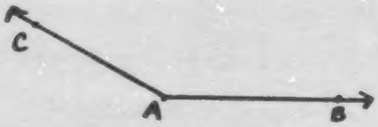
Repeat this procedure for a right angle, an acute angle and an obtuse angle.

Ask the students to measure each angle with a protractor. Write the following statements on the chalkboard.

1) A straight angle measures 180°.
2) A right angle measures 90°.
3) An acute angle measures less than 90°.
4) An obtuse angle measures more than 90° but less than 180°.

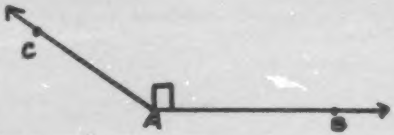
STRATEGY 2: Draw an obtuse angle on the chalkboard.

e.g.,



Ask the students to use a textbook or a sheet of paper and place it over the angle.

e.g.,

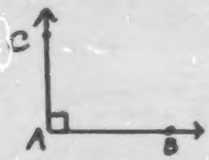


Say, "What kind of angle is in the corner of the book paper?" Elicit that the corner of the book or paper is a right angle. Ask a student to define a right angle. Elicit that a right angle measures exactly 90°. Ask a student to use his book to measure the angle on the chalkboard. Elicit that the angle is greater than 90°. Ask a student to give the name of an angle that is greater than 90°. (Obtuse angle) Repeat this procedure for an acute angle, and a straight angle.

7.09.01.02 **define and identify adjacent, supplementary and complementary angles**

STRATEGY 1: Distribute a protractor to each student or group of students. Draw a right angle on the chalkboard. Ask the students to do the same on a paper using a protractor.

e.g.,



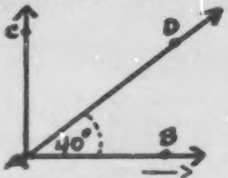
(m)∢ BAC = 90°

7.09.01

STRATEGY 1: cont'd

Construct an angle of 40° on the same vertex.

e.g.,



Name this ray $\overrightarrow{AD}$. Say, "How many angles do we have now?" Elicit there are three angles. Ask a student to name the angles, e.g., ∡ CAD, ∡ DAC, ∡ CAB. Say, "What do all the angles have in common?" Elicit that they have the same vertex. Ask a student what is the relationship of ∡ CAD to ∡ DAB. Elicit that they are next to one another. Explain that two angles with a common vertex and a common ray are called adjacent (next to each other) angles. Ask a student to find the sum of the two angles.

e.g.,

(m) ∡ DAB = 40°

+(m) ∡ DAC = 50°

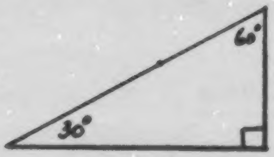
(m) ∡ CAB = 90°

Ask the students to construct a pair of complementary angles. Find the sum. Say that when the sum of the measures of two angles is 90°, the angles are called complementary.

Repeat this procedure for a straight angle. Say that when the sum of the measures of two angles is 180°, the angles are called supplementary. Ask the students to construct their own pair of supplementary angles, name them, and find the sum.

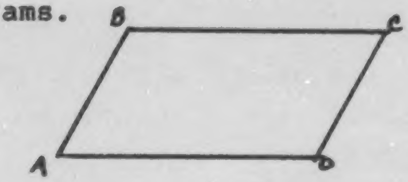
STRATEGY 2: Prepare and distribute triangle cutouts to the students. Make sure that two of the angles add up to 90°.

e.g.,



Ask the students to tear off the 60° angle and the 30° angle, and place them adjacent or next to each other. Elicit that the angle formed is a 90° or right angle. Say that two angles whose sum is 90° are called complementary angles. Distribute parallelograms.

e.g.,



Repeat the above procedure tearing off two angles on the same side of the parallelogram. Explain that two angles adjacent to each other, whose sum is 180°, are called supplementary angles. Say, "Can the complement of an angle be obtuse? Can you name the supplement of an acute angle?"

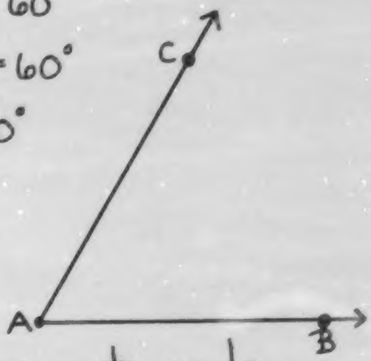
P.O. 7.09.01.01

NAME: ______________________ CLASS: ________ DATE: ________

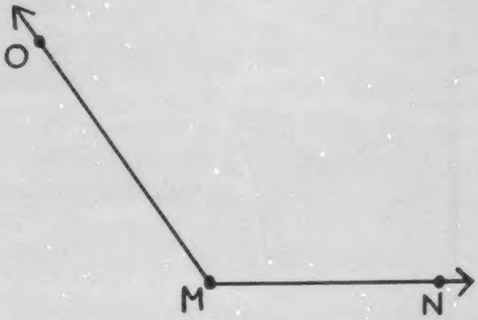
Measure and name each angle. Choose only one way to name each angle. Use your protractor.

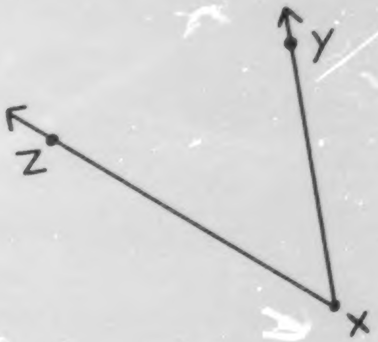
What's Your Angle?

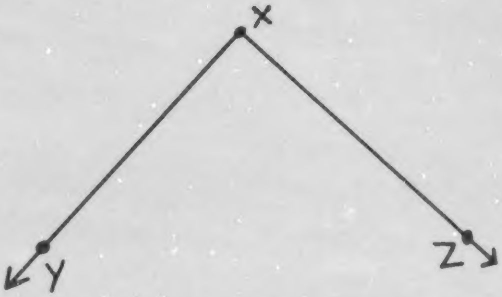
(m)∡CAB = 60°
or
(m)∡BAC = 60°
or
(m)∡A = 60°



∡CAB is an acute angle.







Construct three angles: acute, right, obtuse.
(Label, measure and name each angle.)

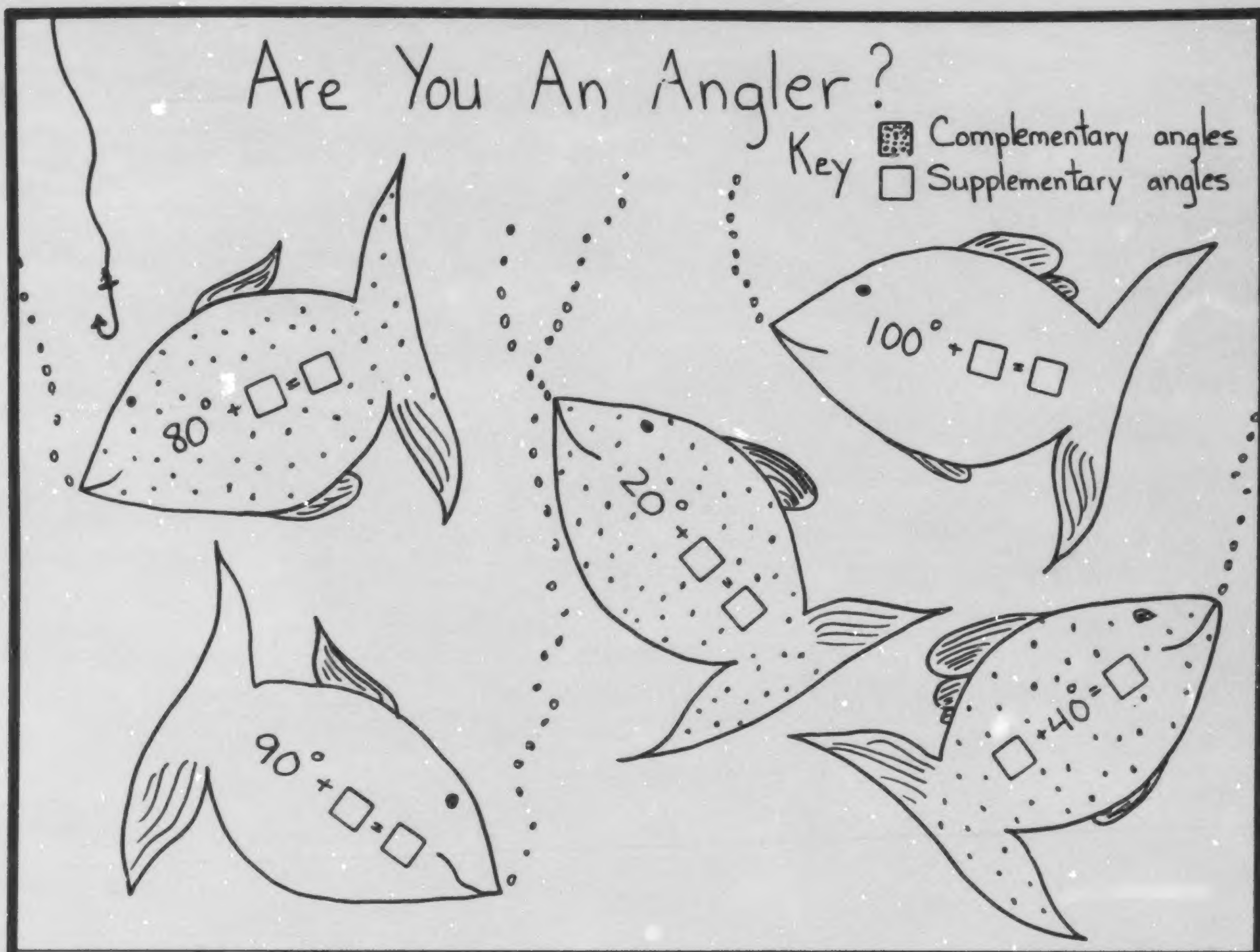
P.O. 7.09.01.02

NAME: ______________________ **CLASS:** __________ **DATE:** ________

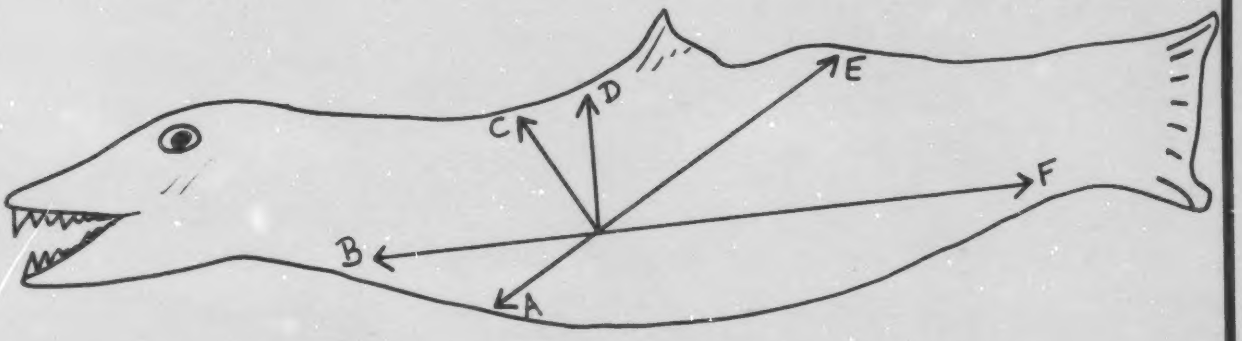
Name the missing angle.

Are You An Angler?

Key
▨ Complementary angles
☐ Supplementary angles



To catch the shark, label, measure and list as many supplementary ∢s and complementary ∢s you can find in the diagram.



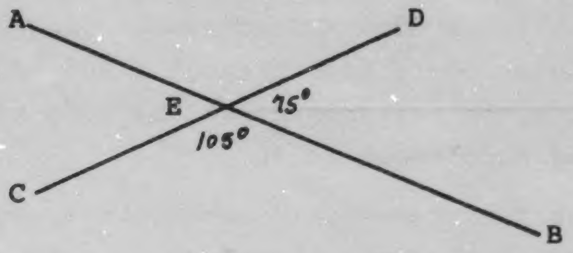
7.09.02 VERTICAL ANGLES

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

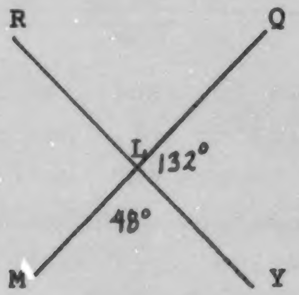
7.09.02.01A use the results of measurements to identify vertical and supplementary angles;
e.g.,



∡'s AED and CEB are vertical angles.
∡'s AEC and CEB are supplementary angles.

7.09.02.02A use the generalization that when two lines intersect, vertical angles are equal.
e.g.,

Find the measures of the missing angles in the diagram.



∡ RLM = 132°
∡ MLY = 48°

7.09.02

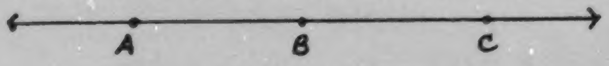
VOCABULARY

TERMS	SYMBOLS
vertical angles congruent	

GETTING STARTED

Draw a straight angle on the chalkboard and label it.

e.g.,

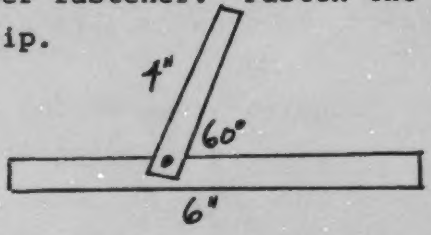


Ask the students to copy the diagram using point B as a vertex. Ask the students to construct a 40° angle with their protractors, and give the measurement of the supplement of this angle. Elicit that the supplement is 40°.

Ask the students to check the measurement with their protractors. Elicit that these angles are supplementary. Repeat this procedure with other diagrams.

Distribute one 6" and one 4" long strips of oak tag cardboard or paper and paper fastener. Fasten the strips together at the center of the 6" strip.

e.g.,



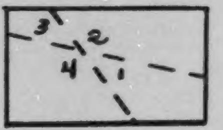
Ask the students to rotate the strips until one side forms a 60° angle (use a protractor). Ask the students to find the measure of the supplement to the 60° angle. Elicit that it measures 120°. Repeat this procedure with other pairs of supplementary angles. Ask a student to define supplementary angles. Elicit that the definition is a pair of angles whose sum is 180°.

7.09.02

7.09.02.01 **use the results of measurements to identify vertical and supplementary angles**

STRATEGY 1: Distribute a sheet of paper. Ask the students to fold their papers twice and mark the angles formed as indicated by the diagram on the chalkboard.

e.g.,

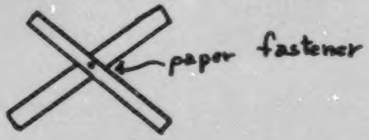


Say, "Carefully tear your paper along each crease. Place angles 1 and 3 on top of each other; then do likewise with angles 2 and 4." Ask several students to describe the relationship of these pairs of angles.

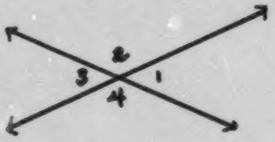
Ask the students to reconstruct their paper so as to form the original sheet. Elicit from the students that opposite angles have the same measure. Say that opposite angles formed by intersecting lines are called vertical angles. Ask a student to use the diagram on the chalkboard and name a pair of supplementary angles, e.g., 1 and 2, 2 and 3, 3 and 4, 4 and 1. Elicit that a pair of intersecting lines form four pairs of supplementary angles.

STRATEGY 2: Distribute to each student a protractor and two oaktag strips approximately 6" by $\frac{1}{2}$".

e.g.,



Draw two intersecting lines on the chalkboard.



STRATEGY 2: cont'd

Ask the students to arrange the strips as illustrated on the chalkboard. Say, "Measure the four angles formed by your strips, and record your measurements in the following way." Write the following chart on chalkboard.

Angle	Measure of Angle
∡ 1 =	
∡ 2 =	
∡ 3 =	
∡ 4 =	

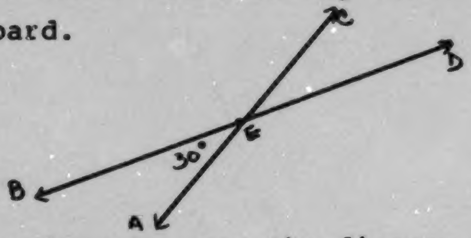
Ask a student for his/her measures for ∡ 1 and ∡ 3. Ask another student to give his/her measures for ∡ 2 and ∡ 4. Ask other students to say the measures of their angles. Elicit from the students that ∡ 1 and ∡ 3 or ∡ 2 and ∡ 4 have the same measures regardless of how intersecting strips are arranged. Say, "When two straight lines intersect, the opposite angles formed are called vertical angles. Vertical angles always have the same measure." Emphasize that angles whose measures are the same are called congruent angles.

Ask the students to find the sum of the measures for angles 1 and 2, and for angles 3 and 4. Elicit the results. Say, "When the sum of the measures of two angles is 180°, the angles are called supplementary angles." Ask a student to name other supplementary angles in the diagram.

7.09.02

7.09.02.02 **use the generalization that when two lines intersect, vertical angles are equal**

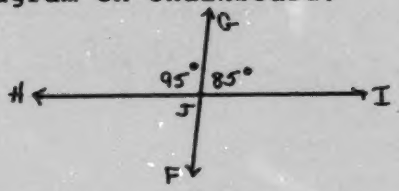
STRATEGY 1: Draw and label intersecting lines on the chalkboard.
e.g.,



Ask the class to copy the diagram into their notebooks.

Ask a student for the measure of ∡ AEB. Elicit from the students that vertical angles have equal measures. Explain to the students that equal angles are called congruent angles. The symbol for congruent is ≅ .

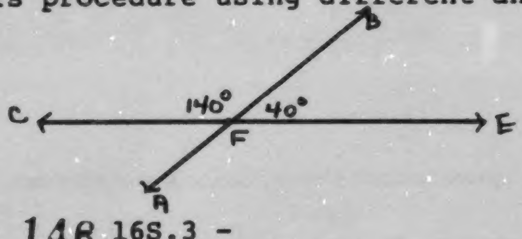
Draw this diagram on chalkboard.
e.g.,



Ask the students to copy the diagram and fill in the measures of the other angles.

STRATEGY 2: Ask the class to name the vertical angles formed by the intersecting lines. Review the definition of vertical angles and congruent angles.
e.g., ∡ FJI ≅ 85° ∡ FJI ≅ ∡ HJG

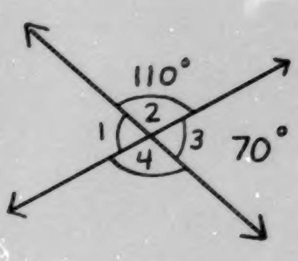
Repeat this procedure using different angle measures.
e.g.,

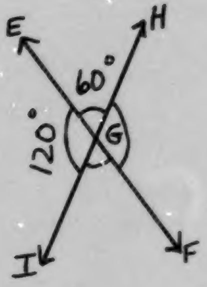


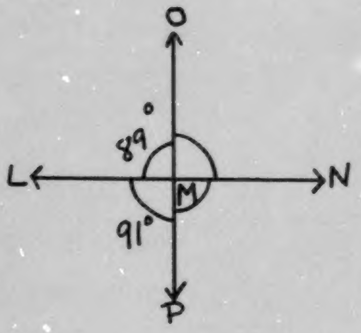
P.O. 7.09.02.01-02

NAME: ____________________ CLASS: ________ DATE: ________

1. Name the vertical and supplementary angles.



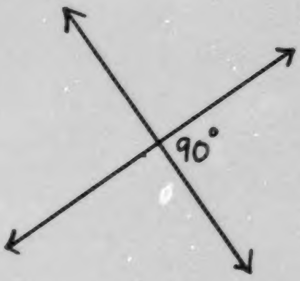


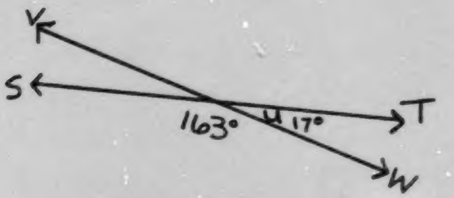


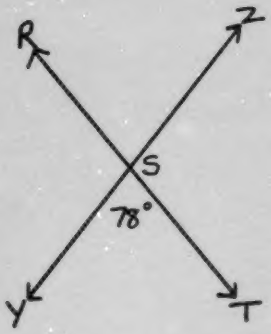
Vertical: Vertical Vertical:

Supplementary: Supplementary: Supplementary:

2. Find the measures of the missing angles in each diagram.







7.09.03 TRIANGLES

(3 lessons)

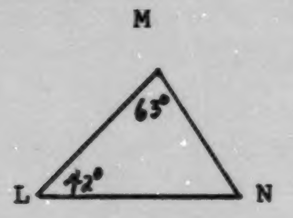
PERFORMANCE OBJECTIVES

The student will be able to...

7.09.03.01A solve numerical problems involving the angles of a triangle;

e.g.,

Find the measure of the third angle.



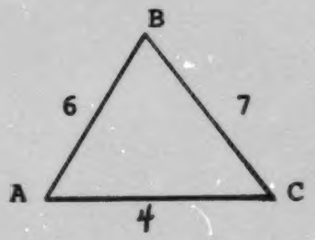
$\measuredangle N = 180° - (63 + 42)°$

$\measuredangle N = 180° - 105°$

$\measuredangle N = 75°$

7.09.03.02A determine that the sum of the lengths of any two sides of a triangle is greater than the length of the third side;

e.g.,



$AB + BC > AC$ $(6 + 7 > 4)$

$AB + AC > BC$ $(6 + 4 > 7)$

$BC + AC > AB$ $(7 + 4 > 6)$

7.09.03.03A solve verbal problems related to skills learned in this module.

e.g.,

Can a triangle be constructed whose sides are 12 ft., 7 ft., and 4 ft. long?

No, because 7' + 4' = 11' and 11' is smaller than 12'.

7.09.03

VOCABULARY

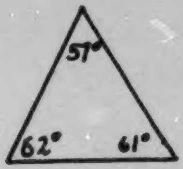
TERMS

triangle

GETTING STARTED

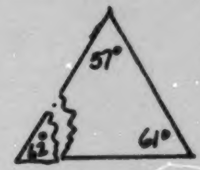
Prepare and distribute to each student a protractor and triangle cutouts. The triangles should be different sizes. Ask the students to measure each angle of the triangle and label it.

e.g.,

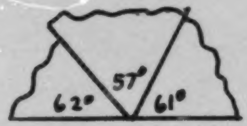


Ask the students to tear off each angle.

e.g.,



Then ask the students to line up each angle.



Say, "What is the measure of this large angle?" Elicit 62° + 61° + 57° = 180° and measure with the protractor. Say, "What does this tell us about the sum of the angles of a triangle?" Elicit that the sum of the angle of a triangle is 180°.

Draw three triangles on the chalkboard.

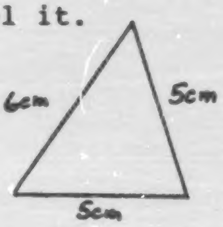
e.g.,

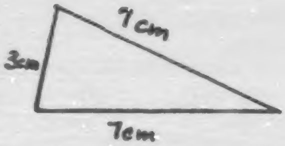
7.09.03

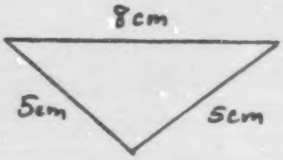
GETTING STARTED cont'd

Ask three students to come to the chalkboard and measure each side and label it.

e.g.,





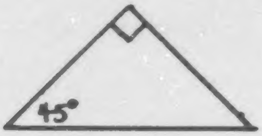


Ask the students how to find the perimeter of a triangle. Elicit that we add each side. Ask three other students to come to the chalkboard and find the perimeter of each triangle.

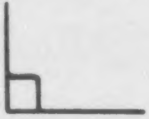
7.09.03

7.09.03.01 **solve numerical problems involving the angles of a triangle**

STRATEGY 1: Draw the following diagram on the chalkboard.
e.g.,



Ask a student to name the measure of angle C without a protractor. Say, "How do you know?" Elicit that 90° angles are always marked by a box.
e.g.,



Ask the students how they would find the measure of angle C. Elicit the following steps.

$\angle C = 180^\circ - (90 + 45^\circ)$

$\angle C = 180^\circ - 135^\circ$

$\angle C = 45^\circ$

Repeat this procedure with other triangles that have missing angles.

STRATEGY 2: Write on the chalkboard.
e.g.,

$\angle R = 60^\circ$

$\angle S = 40^\circ$

$\angle T = \square^\circ$

Ask a student how he/she would find the missing angle. Elicit that the sum of the angles of any triangle is equal to 180°, and therefore, we would add the two known angles, 60 + 40, and subtract their sum from 180°.

7.09.03

STRATEGY 2: e.g.,
cont'd

180 - (60 + 40) = ☐
180 - 100 = ☐
80°

Repeat this procedure with other triangles.

e.g.,

∢ M = 65°
∢ N = 30°
∢ O = ☐

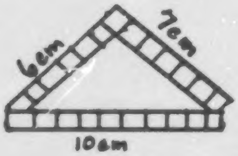
The sum = 180°

Find the missing angle.

7.09.03.02 **determine that the sum of the lengths of any two sides of a triangle is greater than the length of the third side**

STRATEGY 1: Distribute to the students 4 cm, 5 cm, 6 cm and 10 cm strips of paper. Ask the students to make triangles from the different strips. Ask each student to list the length of the three sides of each triangle.

e.g.,



Side 1	Side 2	Side 3
6	7	10

Ask the students to find the sum of any two sides

STRATEGY 1: con't

of the triangle. Ask if the sum is greater than the measure of the third side.

e.g.,

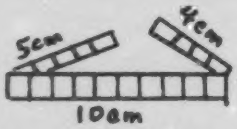
a) 6 + 7 > 10

b) 7 + 10 > 6

c) 6 + 10 > 7

Ask the students to try to construct a triangle with 4 cm, 5 cm and 10 cm strips.

e.g.,



After experimenting the students will see that this is impossible. Say, "Why is this impossible?" Elicit that the sum of any two sides of a triangle must be greater than the third side. Repeat this procedure using other triangles.

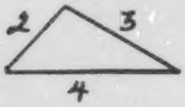
STRATEGY 2: Prepare centimeter strips of various lengths, e.g., 5 cm, 7 cm, 10 cm, 2 cm, 3 cm and 6 cm. Ask several students to come to the chalkboard and construct a triangle using any three strips. Ask each student to add every pair of sides.

e.g.,

3 + 4 = 7

4 + 2 = 6

3 + 2 = 5



Ask the students to see if the sum of each pair of sides is greater than the third side.

e.g.,

7 > 2,   6 > 3,   5 > 4

Say, "What can we say about the sum of any two sides of a triangle?" Elicit that it must be longer than the third side. Repeat this procedure

STRATEGY 2: cont'd

using the 2 cm, 3 cm, and 5 cm strips. Say, "Why is it impossible to construct this triangle?" Elicit that 2 + 3 is not greater than 5.

7.09.03.03 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following problem on the chalkboard.

> Steve was asked to construct a triangle whose sides were 15 feet by 8 feet by 5 feet. After working for a while Steve threw up his hands and said, "It's impossible!" Is Steve right?

Elicit from the students that Steve is right. Ask a student to explain.

e.g., 8 ft + 5 ft = 13 ft and 13 ft is less than 15 ft.

Repeat this with sets of numbers. Ask a student to list the measurements that would make a triangle.

STRATEGY 2: Write the following problem on the chalkboard.

> A carpenter is constructing the frame of a triangular roof. If he has cut two boards of 7 feet and 4 feet, what are the possible third sides he can cut?

Ask a student to read the problem aloud. Say, "How can we set up an example to solve this problem?" Elicit that we add the two sides and determine that the third side can be any length longer than the sum.

e.g., 7 feet + 4 feet = 11 feet. Therefore, the third side can be 12 feet, 18 feet, 21 feet...

Repeat this procedure using other verbal problems.

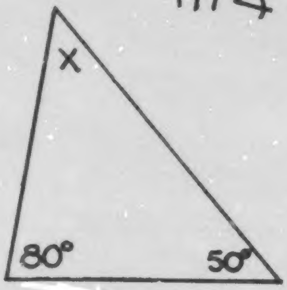
P.O. 7.09.03.01

NAME: ______________________ CLASS: ________ DATE: ________

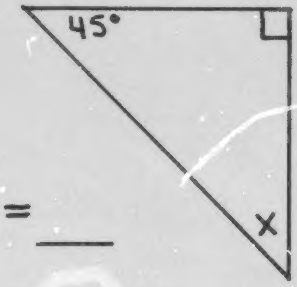
Find the measure of angle X in each diagram without a protractor.

1.

m∡X= ___

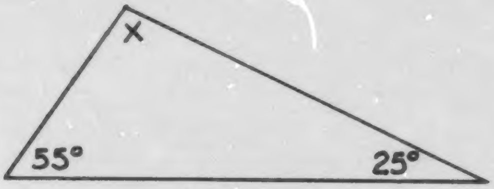


2.



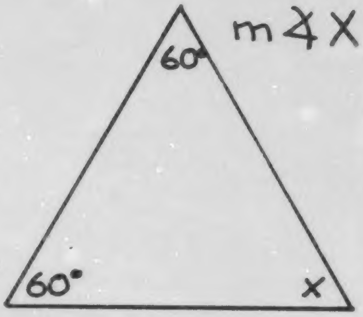
m∡X= ___

3.



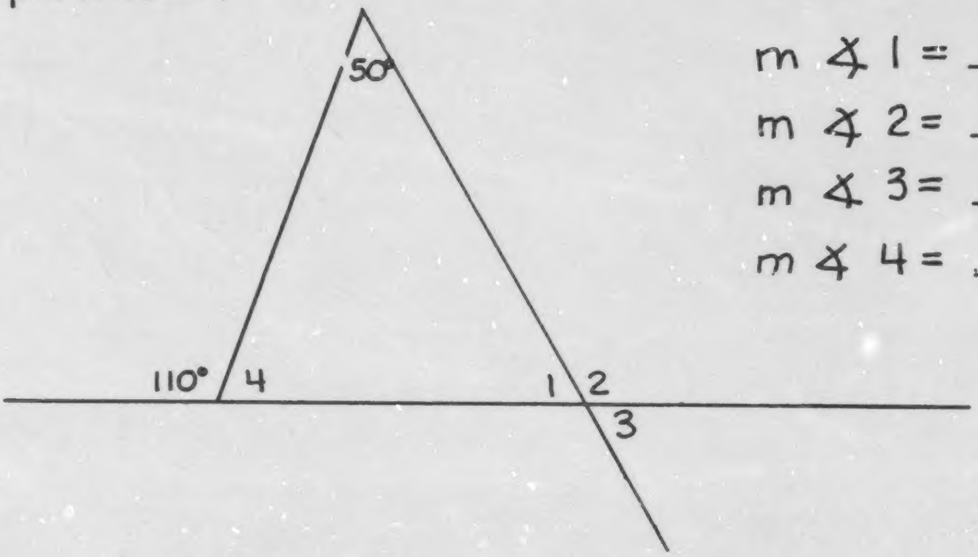
m∡X = ___

4.



m∡X = ___

5. Find the measure of each angle without a protractor.



m∡1 = ___

m∡2 = ___

m∡3 = ___

m∡4 = ___

P.O. 7.09.03.02

NAME: ______________________ CLASS: __________ DATE: ________

I. Is it a triangle?
II. Find a possible third side.

I. 7, 12, 20	6, 8, 10
8, 11, 9	4, 6, 1
12, 9, 4	3, 9, 6

II.

$1\frac{1}{2}$, $2\frac{1}{2}$, △

8.2, 6.1, △

16, 11, △

6, 9, △

P.O. 7.09.03.03

NAME: ______________________________ CLASS: ________ DATE: ________

Solve the following problems.

1. The diagonal of a square measures twelve inches. What is the smallest length the side can be?
(Hint: Draw a diagram.)

2. The height of a parallelogram is 8 inches the base is 14 inches. What is the least the diagonal can be?

3. The height of a T.V. screen is 12 inches, the diagonal of the screen is 13 inches. What is the smallest length the base of the T.V. set can be?

4. Can you construct a triangle whose sides are 8", 4", 2"? Explain. (Experiment with strips of paper.)

7.07.01 DECIMAL RELATIONSHIPS

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.07.01.01A extend the base ten place value chart to include decimals whose denominators are powers of 10, and read and write decimal fractions in numerical and word form; e.g.,

.5 6 2 ⑧

Write the place value of the circled digit. (ten-thousandths)

Write the face value of the circled digit. (8)

Write the value of the circled digit. (eight ten-thousandths)

7.07.01.02A identify equivalent decimals; e.g.,

.0004 = .00040 = .000400

7.07.01.03A compare and order decimals;

e.g.,

.0028 > .002

3.0008, 4.0001, 3.0002	unordered
3.0002, 3.0008, 4.0001	ordered from smallest to largest

7.07.01.04A round off a given decimal to specified place;

e.g.,

Round .6472 to the nearest tenth, hundredth, thousandth (.6, .65, .647)

7.07.01.05A solve verbal problems related to skills learned in this module.

e.g.

Alan's batting average is .3725 and Sam's is .3723. Round both averages to the nearest thousandth and determine who has the higher batting average.

.3725 rounds to .373

.3723 rounds to .372

Therefore, Alan has the higher batting average.

VOCABULARY

TERMS

place value
decimal
equivalent

GETTING STARTED

On the chalkboard write the number 8456. Ask the students to say the number aloud, e.g., 8 thousand, 4 hundred, fifty six.

Say, "What is the place value of 8? of 4? of 5? of 6?"

Ask the students to write 8456 in expanded notation.

e.g., (8 x 1000) + (4 x 100) + (5 x 10) + (6 x 1).

Say that each place value is 10 times the value of the place to its right and $\frac{1}{10}$ the value of the place to its left.

Divide the class into two groups, A and B. Prepare 36 cards with random 1-, 2-, 3- and 4-digit numbers written on them, one number to a card. Shuffle the cards. Ask the first player from team A to choose one card, e.g., 386 . The first player from team B chooses one card, e.g., 63 . The team with the larger number gets a point. Continue playing until one team reaches 10 points.

7.07.01

7.07.01.01 **extend the base ten place value chart to include decimals whose denominators are powers of 10, and read and write decimals fractions in numerical and word form**

STRATEGY 1: Ask the students if there exists numbers less than 1. Elicit examples from the students such as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$,...then ask the students what the smallest place value is that they know. Elicit that it is the ones place. Then ask the students if the ones place can be used to express numbers less than 1. Elicit that we will have to extend our base ten place values so that we will be able to express numbers less than 1. Say that we learned that each place value in our system is 10 times the value of the place to its right and $\frac{1}{10}$ the value of the place to its left. Say that we will have to extend this place value to accommodate numbers which are less than one, and that we place a point, called a decimal point, to the right of the ones place to show numbers less than 1. Ask the students what we would call the place to the right of the ones place.

Elicit that it must be $\frac{1}{10}$ or the tenths place. Similarly the next place to the right must be $\frac{1}{10}$ of the tenths place, or the hundredths place. Repeat this procedure to find the other place values to the right, e.g., thousandths, ten-thousandths. Write .34 on the chalkboard. Say, "We read this number as thirty-four hundredths." Write .356 on the chalkboard. Ask a student to read the number aloud, e.g., three hundred fifty-six thousandths. Write forty-eight hundredths on the chalkboard. Ask a student to write the number.

STRATEGY 1: cont'd

e.g., 48

Repeat these procedures using decimals in word and numerical form.

STRATEGY 2: Prepare a place value chart including decimals or draw on the chalkboard.

e.g.,

1000	100	10	1	.1	.10	.100
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Say, "How do the decimal place value columns follow the whole number system?" Elicit that each place to the left is 10 times the place to its right and each place to the right is $\frac{1}{10}$ of the place to its left. Write 36.28 on the chalkboard. Ask a student to write each number in its proper place value column.

e.g.,

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		3	6	2	8	

Say, "Why do we use the decimal point?" Elicit that we use the decimal to separate the whole number from the decimal. Ask a student to read the decimal aloud. Repeat this procedure using other decimals.

e.g.,

0.52, 5.72, 7.283.

Write fifteen and eight hundredths on the chalkboard. Ask a student to write the decimal in numerical form on the chalkboard, e.g., 15.08.

Repeat this procedure using decimal numbers written in word and numerical form.

7.07.01.02 identify equivalent decimals

STRATEGY 1: Write on the chalkboard, .6, .60, .600. Ask the students to read these decimal fractions. Then ask the students to write down the decimals in word form and in fractional form.

e.g.,

$$.6 = 6 \text{ tenths} = \frac{6}{10}$$

$$.60 = 60 \text{ hundredths} = \frac{60}{100}$$

$$.600 = 600 \text{ thousandths} = \frac{600}{1000}$$

Ask the students if there are any commonalities amongst $\frac{6}{10}$, $\frac{60}{100}$, $\frac{600}{1000}$.

Elicit that $\frac{6}{10} = \frac{60}{100} = \frac{600}{1000}$ and therefore .6 = .60 = .600. Say, "We call those equivalent decimals." Say that when we add 0's to the right of the 6 in the decimal .6, we do not change the value of the decimal. Repeat this procedure using .81, .810, .8100.

STRATEGY 2: Draw a decimal place value chart on the chalkboard.

e.g.,

. Tenths	Hundredths	Thousandths	Ten thousandths

Ask several students to write .8, .80 and .800 in the place value chart. Say, "What is the same about all three decimals?" Elicit that each has an 8 in the tenths place. Ask a student to fill in zeros in all other places.

e.g.,

. Tenths	Hundredths	Thousandths	Ten thousandths
. 8			
. 8	0		
. 8	0	0	

7.07.01

STRATEGY 2: cont'd

Say, "Have we changed the value of the decimals by adding these zeros? What can we say about these decimals?" Elicit that they are equivalent. Repeat using .21, .21, .2100.

7.07.01.03 **compare and order decimals**

STRATEGY 1: Say, "Marie is very hungry. She has a choice of 2 slices of turkey. One is .6 of an inch thick and the other is .57 of an inch thick. Which one should she choose to get the thicker one?" Ask the students how they would change the fractions in order to compare more easily. Elicit that they can write .6 as .60 without changing its value. Say, "Which is larger, .60 or .57?" If the students are having difficulty utilize whole numbers to reinforce this concept.

e.g.,

Which is larger, 60 or 57? List the rules for comparing decimals.

e.g.,

(1) If the number of places are the same, compare each digit, .43 .67

(2) If the number of places are different, add zeros to make the number of places the same and compare, e.g., .8 .36 = .80 .36.

Say and write on the chalkboard, "Order the following decimals: .4, .44, .404."

Say that to solve this problem, we must first make the following conversions, .4 = .400, .44 = .440, .404 = .404

Say that we must compare the numbers 400, 440, 404 to get the correct ordering from smallest to largest.

STRATEGY 1: cont'd

e.g.,

.400, .404, .440 or .4, .404, .44.

Repeat using other decimals to compare and order.

STRATEGY 2: Write "The Class 7-210 Record Holder" on the chalkboard. Say, "Make believe we had a race in our class and the top three times were ran by Bill -1.025, Jerry - 1.127 and Juan - 1.003. Who would be our class record holder?" Ask how the students would compare the numbers. Elicit that the comparison is the same as for whole numbers.

e.g.,

1025, 1127 and 1003 and Jaun's time of 1.003 is the fastest.

Repeat this procedure using .637 and .062, 6.002 and 6.020, 0.5 and 0.49.

Say that if the number of places are different, we can add zeros to a decimal to equalize the number of places without changing the value of the decimal.

7.07.01.04 **to round off a given decimal to a specified place**

STRATEGY 1: Say, "Ivan had to thread a piece of rope through a large industrial needle. The rope's diameter measured .68 of an inch. To know whether or not it would fit through the eye of the needle, Ivan had to know the diameter of the rope correct to the nearest tenth of an inch. How could he find this out?" Elicit that .68 is between .60 and .70, or between .6 and .7.

Ask the students whether .68 is closer to .60 (.6)

STRATEGY 1:
con't

or to .70 (.7). Elicit that it is closer to .70, and when .68 is rounded off to the nearest tenth of an inch the answer is .7 of an inch.

Repeat this procedure using other problems.

Ask the students to state a general procedure for rounding off a decimal to a given number of places. Elicit that we must first write one more digit than is asked for by the problem and omit the rest of the digits, e.g., if we are to round 3.6273 to the nearest tenth, we write 3.62. Then we look at the last digit. If it is less than 5 we omit the digit.

e.g.,

3.62 would round to 3.6

If it is greater than or equal to 5 we drop the digit and add 1 to the digit to its left.

e.g.,

3.68 would round to 3.7

Ask the students to round 3.7845 to the nearest hundredth, 16.82 to the nearest tenth and 7.8333 to the nearest thousandth. Repeat this procedure by rounding other decimals.

STRATEGY 2:

Say and write the following on the chalkboard.

Would it be cheaper to buy 10
apples at $1.06 or to buy 10 apples,
one at a time?"

Ask the students how they would solve this problem. Elicit that we would need to calculate the cost of one apple by dividing.

e.g.,

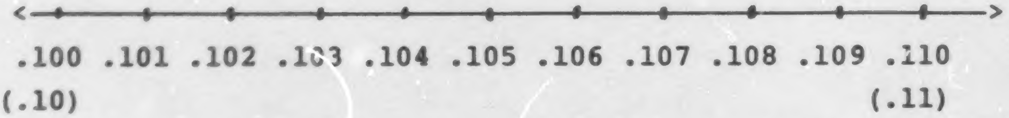
$$10\overline{)1.060}\quad .106$$

Say, "Would the store charge $.106?" Elicit that we would have to round to the nearest hundredth.

STRATEGY 2: cont'd

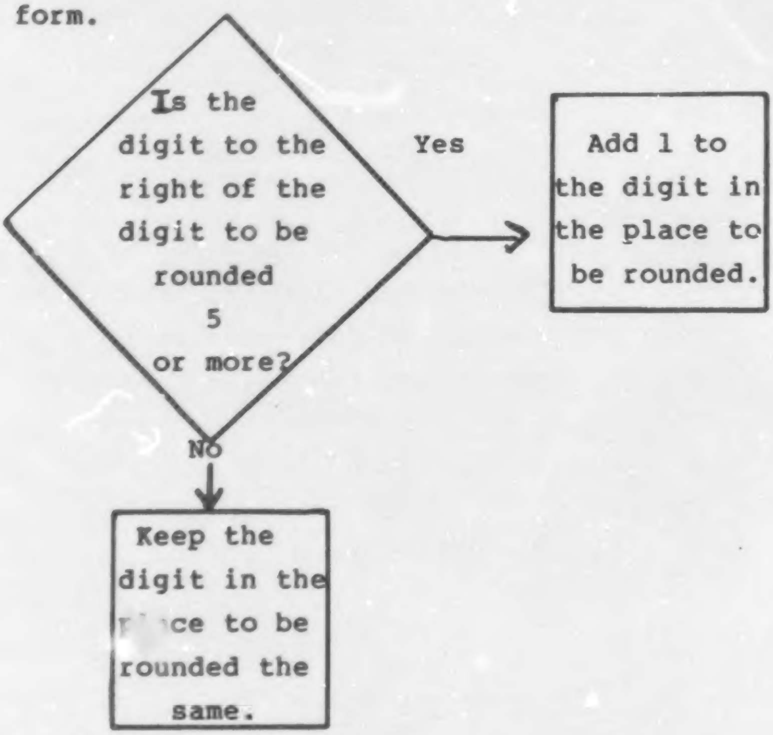
Draw a number line on the chalkboard.

e.g.,



Say, "Is .106 closer to .10 or .11?" Elicit that .106 is closer to .11. Say that the storekeeper would charge .11 per apple. Say, "At that cost, how much would 10 apples be?" Elicit .11 x 10 = 1.10 which is more expensive than $1.06.

Say, "Let's look at the process of rounding and draw a chart to tell us round. Which digit helps us to decide?" Elicit that the digit to the right of place to be rounded is the helping digit. Say, "What numbers are significant?" Elicit less than 5 or 5 or more. Write the following in diagram form.


Is the digit to the right of the digit to be rounded 5 or more?
Yes
Add 1 to the digit in the place to be rounded.
No
Keep the digit in the place to be rounded the same.

Ask the students to use the chart to round decimals to specific places.

e.g.,

Round 6.184 to the nearest hundred.

7.07.01.05 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following on the chalkboard.
e.g.,

> A mechanic has two nails. One is .4253 inches long, and the other is .4257 inches long. Which nail is larger?

Elicit that we have to compare .4253 and .4257. Ask what is the technique we have to do this? Elicit that we can use rounding. Say that we can round .4253 to .425 and round .4257 to .426. Ask, "Which is the larger nail?" Elicit .426 is the larger nail.

Repeat this procedure using other verbal problems.

STRATEGY 2: Write the following decimals on the chalkboard, 12.75, 11.001, 4,567, 2.0502. Ask the students to take out a piece of paper and write a word problem which involves rounding to whatever decimal place they would like. Ask the students to give their problems to a neighbor to solve. Continue this procedure until each student has had practice in solving a verbal problem.

P.O. 7.07.01.01

NAME: ______________________ CLASS: ________ DATE: ________

Cross out the correct place value.

0.03 3 ~~3~~	three thousandths
4 4.4 0 4	four tenths
3 3 3.3 3	three hundredths
0.8 8 8 8	eight ten thousandths
5 5.5 5 5	five thousandths
0.9 0 9 9	nine ten thousandths
6 6 6.6 6	sixty-six hundredths
1.0 1 1 0	one hundredth
0.7 7 7 7	seven tenths

P.O. 07.07. 01.02

NAME:______________________________CLASS:________DATE:________

Draw a line from the decimal in column I to its equivalent in column II.

① .8
② .07
③ .57
④ .082
⑤ .825
⑥ .0700
⑦ .0850
⑧ .00500
⑨ .15
⑩ .210500

ⓐ .07
ⓑ .82500
ⓒ .005
ⓓ .1500
ⓔ .800
ⓕ .07
ⓖ .570
ⓗ .08200
ⓘ .2105
ⓙ .085

P.O. 7.07.01.03

NAME: ________________________ CLASS: ________ DATE: _______

I. Circle the larger number.

a. .7, .721, .7218

b. .8, .78, .718

c. .9, 1.0, .899

d. 3.01, 3.10, 3.001

e. .3, .033, .0033

II. Order these numbers from smallest to largest.

a. .0025, .005, .0505

b. 2.9, 3.101, 2.19

c. 8.142, 9.0012, 8.0142

d. 5.003, 6.001, 5.008

e. 8.04, 9.004, 7.99

P.O. 7.07.01.04

NAME: ______________________ CLASS: __________ DATE: ________

Round off each number. Find the answer. The number in front of the answer tells you what letter to put in the box.

Ⓞ	.64 to the nearest tenth	①	.84
Ⓣ	.286 to the nearest hundredth	②	.6
Ⓘ	.6445 to the nearest thousandth	③	.76
Ⓔ	.4154 to the nearest thousandth	④	.29
Ⓝ	.762 to the nearest hundredth	⑤	.995
Ⓗ	.9946 to the nearest thousandth	⑥	.645
Ⓜ	.8363 to the nearest tenth	⑦	.836
Ⓓ	.8363 to the nearest hundredth	⑧	.8
Ⓣ	.8363 to the nearest thousandths	⑨	.415

1	2	3	4	5	6	7	8	9

WHAT DID THE GOAL POST SAY TO THE FOOTBALL?

P.O. 7.07.01.05

NAME: ______________________________ CLASS: _________ DATE: ________

Solve.

1. John's temperature was 102.8.

   Round off the temperature to the nearest degree.

2. Triangle A has area of 13.954 square inches.

   Triangle B has area of 13.956 square inches.

   Which triangle has a larger area?

3. Boy Scout troop T walked 10.342 miles in the woods.

   Troop R walked 10.347 miles.

   Which troop walked the smaller number of miles?

7.07.02 ADDITION OF DECIMALS

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.07.02.01A compute the sum of two or more decimals arranged horizontally;
e.g.,

$$34.007 + .958 + 2.347 + 20.1 = 57.412$$

7.07.02.02A solve verbal problems related to skills learned in this module.
e.g.,

Judy wrote checks for $42.30, $17.56, $8.91 and $32 one day. How much did she spend that day?

$$\begin{array}{r} \$42.30 \\ 17.56 \\ 8.91 \\ \underline{32.00} \\ \$100.77 \end{array}$$

Judy spent $100.77 that day.

7.07.02

VOCABULARY

TERMS

decimal point
column

GETTING STARTED

Write 6 or 7 numbers on the chalkboard, varying the number of digits in each, e.g., 23, 438, 72, 6,043, 25, 735.

Ask the students how they would arrange the numbers in order to add them. Elicit that in order to add these numbers we have to line them up vertically with each number in the proper place value column.

e.g.,

```
   23
  438
   72
6,043
   25
  735
```

Repeat this procedure using other examples.

Ask the students to play an addition game. Say that the way to play this game is to start off with a number, e.g., 47 and have each student add 4 to the number, e.g., 47+4, then 5, e.g., 47+5, then 6, e.g., 47+6,... until all students have had a turn. Continue for five minutes changing the starting number each time.

7.07.02

7.07.02.01 **compute the sum of two or more decimals arranged horizontally**

STRATEGY 1: Place the following numbers on the chalkboard, $2.38, $4.57, $8.16, $12.14 and ask the students how to add up these numbers. Elicit that adding decimals is the same as adding whole numbers except for the decimal point. Say that you line up the whole numbers using place value. Decimals line up to the right of each decimal point, vertically. Write this example on the chalkboard.

e.g.,

```
  $ 2.36
  $ 4.57
  $ 8.16
+ $12.14
```

Elicit that when we line up all of the ones place values, we automatically line up the decimal points as well.

Write on the chalkboard and ask the students how they would add the numbers.

e.g.,

.078, 3.14, 6.32, 40.007

Elicit that .078 doesn't have a ones place value digit and therefore we can't line it up using the ones place value. Say that all the numbers have decimal points and, as we discovered in the last problem, all the decimal points must also line up. Say that we line up the decimal points first and then transcribe the numbers depending on their relation to the decimal point.

e.g.,

```
    .078
   3.14
   6.32
+ 40.007
```

177

STRATEGY 1: cont'd

Say that we may add zeros to the left if we wish to make all the numbers have the same amount of digits.

e.g.,

```
   00.078
   03.140
   06.320
 + 40.007
```

STRATEGY 2:

Write the following problem on the chalkboard.

```
    .007
   3.1
 +200.3
```

Ask the students if they think the problem is set up correctly? Ask the students to state what the "7" in the first number represents. Elicit that it represents 7 thousandths. Ask the students what the "1" in the second number represents? Elicit that it represents 1 tenth. Ask, "Do you think the 7 and the 1 should be in the same column?" Elicit that because they stand for different place values, they must be in different columns. Say that the purpose of the decimal point is to separate whole numbers from decimal fractions. Write a chart on the chalkboard with the problem rewritten in it.

e.g.,

.	t	h	th
.	0	0	7
3.	1		
200.	3		

Say that the decimal points must always be aligned when adding or subtracting, because when we add we always add like units. Write several decimals on the chalkboard in a horizontal arrangement, and ask the students to add them. Check and repeat this procedure using other examples.

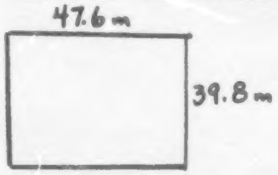
7.07.02

7.07.02.02 Solve verbal problems related to skills learned in this module

STRATEGY 1: Write on the chalkboard.

The Jones' are putting a fence around their yard. The yard is 47.6 meters by 39.8 meters. How many meters of fencing do they need?

e.g.,



Elicit the methods of solving verbal problems using addition of whole numbers. Say, "Adding decimals is just like adding whole numbers, but first we line up the decimal points."

e.g.,

$$\begin{array}{r} 47.6 \\ 47.6 \\ 39.8 \\ +\ 39.8 \\ \hline 174.8 \end{array}$$

174.8 meters of fencing

Ask the students to make up their own verbal problem using decimals. Ask them to exchange problems with each other and write several problems on the chalkboard.

STRATEGY 2: Write the following verbal problems on the chalkboard.

1. A druggist mixes 432 liters of alcohol with 235 liters of water. How many liters are there in the total mixture?

2. A druggist mixes 43.2 liters of alcohol with 23.5 liters of water. How many liters are there in the total mixture?

STRATEGY 2: cont'd

Say, "What is different about the two problems?" Elicit that the problems are identical except that problem 2 has decimals and problem 1 has whole numbers. Elicit that the wording in verbal problems involving decimals is exactly the same as the wording in whole number problems. Ask the students to write out their own verbal problems using decimals.

P.O. 7.07.02.01

NAME:____ CLASS:__________ DATE:__________

Solve the problems below and break the code.

G.	11 + 3.4 + 5.983 =
D.	0.8 + 2.091 + 15.56 =
E.	0.3 + 14 + 9.71 =
O.	10.2 + 5 + 3.64 =
B.	4.56 + 9.4 + 3.768 =
R.	12.2 + 6.78 =
N.	13.6 + 45.84 =

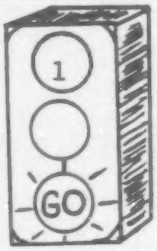
17.728 24.01 20.383 18.84 18.84 18.451 18.84 18.98

17.728 24.01 20.383 18.84 59.44 24.01

P.O. 7.07.02.02

NAME: ______________________ CLASS: ________ DATE: ________

Solve the following verbal problems.



A traingular lot has sides of 14.5 meters, 32.9 meters and 18.7 meters. How many meters of fencing are needed to fence in the lot?

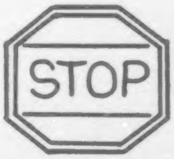
2. Howard wrote checks for $18.65, $17.83, $106.42 and $24.98 one day. How much did he spend that day?

3. A druggist mixed 3.75 liters of alchohol with 7.65 liters of hydrogin peroxide. How many liters were in the resulting mixture.

4. Jose ran the following distances during the week. Mon. 2.3 miles, Tue. 3.1 miles, Wed. 2.8 miles, Thurs. 3.0 miles, and Fri. 3.2 mile How many miles has Jose run so far this week?

5. Morticia Gomez needs the following amounts of material to create her designer shroud. 7.35 yards somber grey and 9.03 yards of chalky white. How many yards of material does she need altogether?

6. Bob Opfel needs the following lengths of wood to finish his project: 13.5cm, 8.35cm, 9.18cm and 12.67cm. In total, how much wood does Bob need?



Unit 17

7.07.03 SUBTRACTION OF DECIMALS

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.07.03.01A compute the difference of two decimals arranged vertically with the minuend having fewer decimal places than the subtrahend; e.g.;

$$4.8 - .396 = \begin{array}{r} 4.8 \\ \underline{-\ .396} \end{array} = \begin{array}{r} 4.\overset{7}{\cancel{8}}\overset{9}{\cancel{0}}\overset{10}{\cancel{0}} \\ \underline{-.396} \\ 4.404 \end{array}$$

7.07.03.02A solve verbal problems related to skills learned in this module.
e.g.,

Lorraine has \$16.86. She spends \$6.29. How much does Lorraine have left?

$$\begin{array}{r} \$16.\overset{7}{\cancel{8}}\overset{16}{\cancel{6}} \\ \underline{-\ \ 6.29} \\ \$10.57 \end{array}$$

GETTING STARTED

Write the following problem on the chalkboard.

e.g.,

6.835 + 17.42 + 2.004 =

Ask the students how they would find the answer to the problem. Elicit that in order to find the sum in a decimal problem, we must first line up the decimal points. Remind the students that if they wish, they may use zeros as place holders.

e.g.,

```
  6.835
 17.420 <—— place holder
+ 2.004
```

Repeat this procedure using other decimal addition problems.

Prepare four cards with a decimal on each.

e.g.,

$\boxed{24.8}$ $\boxed{3.024}$ $\boxed{.341}$ $\boxed{1.27}$

Ask a student to come to the chalkboard and arrange the cards to form an addition example. Ask another student to write the example on the chalkboard. Check to see that the students are aligning their numbers correctly.

```
  3.024         1.27
   .341          .341
  1.27          3.024
+24.8         +24.8
```

If the students are having problems aligning the decimals, suggest making place value columns. The students may add zeros to help in the addition.

e.g.,

```
  3.024
   .341
  1.270 <—— place holder
+24.800
```

Ask the students to solve the problem. Repeat this procedure using other cards.

7.07.03

7.07.03.01 **compute the difference of two decimals arranged vertically with the minuend having fewer decimal places than the subtrahend**

STRATEGY 1: Say, "Mark has eight dollars. How do we write eight dollars?" Elicit that eight dollars can be written many ways. (e.g., \$8, \$8.00, eight dollars, $\frac{8}{1}$ dollars...) Say, "Does \$8.00 mean eight dollars?" Elicit that the decimal point separates the whole dollars from the parts, and \$8.00 tells us that there are eight whole dollars. Write the following problem on the chalkboard.

e.g.,

$$\begin{array}{r} \$8. \\ \underline{-\ 0.64} \end{array}$$

Say, "What does this problem say? (8 dollars less 64 cents) What must be done to subtract? (Two zeros should be added after the decimal point in the eight dollars, e.g., \$8.00.) We use two zeros after the decimal point when we subtract with money. Since we don't have a coin smaller than a penny, or one hundredth of a dollar, we only have to go to the hundredths place. Sometimes we add zeros to the end of decimal fractions. However, adding the zeros doesn't change the values."

Write the following problem on the chalkboard.

e.g.,

$$\begin{array}{r} 4.8 \\ \underline{-.396} \end{array}$$

Explain that adding two zeros after the 4.8 in order to solve the problem is acceptable. Stress that the two zeros do not change the value of the decimal. Ask a student to solve the problem.

e.g.,

$$\begin{array}{r} 4.\overset{7}{\cancel{8}}\overset{9}{\cancel{0}}\overset{10}{\cancel{0}} \\ \underline{-.396} \\ 4.404 \end{array}$$

Repeat this procedure using other decimal subtraction examples.

7.07.03

STRATEGY 2: Write the following problem on the chalkboard.

> Frank brought a looseleaf binder for $2.39, a package of looseleaf paper for $.99, pens for $1.98 and a protractor for $1.27. He used a ten dollar bill to pay for his purchases. He received 3 one-dollar bills, three quarters, one dime, one nickel and two pennies change. Was the change correct?

Ask the students the steps they would use to solve the problem. Elicit the following steps.

1. Find the total cost of the purchases;
2. Find the amount of money that should be received;
3. Determine if the amount of money given was correct.

Ask the students to solve the problem by solving each step.

e.g.,

Step 1:

$$\begin{array}{r} \$2.39 \\ .99 \\ 1.98 \\ \underline{1.27} \\ \$6.63 \end{array}$$

Step 2:

$$\begin{array}{r} \$1\overset{9}{\cancel{0}}.\overset{9}{\cancel{0}}\overset{10}{0} \\ \underline{-6.63} \\ 3.37 \end{array}$$

Step 3: No, Frank didn't receive the correct change.

7.07.03.02 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following verbal problem on the chalkboard.

> A boy ran the 100 meter dash in 15.4 seconds, then later ran the same distance in 13.723 seconds. How many seconds less did he take the second time?

STRATEGY 1: cont'd

Ask a student to read the problem and write the appropriate example on the chalkboard, e.g., 15.4 - 13.723. Ask a student to write the example in vertical form and solve.

Repeat this procedure using other examples.

STRATEGY 2: Write the following menu on the chalkboard.

MENU

Hamburger	$1.90		
Hot Dog	$ .80		
Onion Rings	$ .60		
French Fries	$ .75		
Cola	$ .70	.80	.90

Say, "Tracy has exactly $3.00 and would like to spend it all on lunch excluding the tax." Ask the students to make as many different lunch combinations as they can while trying to use as much of the $3.00 as possible.

e.g.,

Hot dog	$ .80	$3.00	
French fries	.60	-2.30	
Large cola	+.90	$ .70	left
	$2.30		

P.O. 7.07.03.01

NAME: ______________________ CLASS: ________ DATE: ________

Subtract.

① 3.75 - 1.727

② .094 - .08

③ 14.9 - 1.73

④ 100.05 - 26.2

⑤ 8.041 - 3.22

⑥ 52 - 7.06

⑦ 9 - .253

⑧ 6.9 - .472

⑨ 12.02 - 8.119

⑩ 40.08 - 18.7

⑪ 27.63 - 15.008

⑫ 61.3 - .102

P.O. 7.07.01.02

NAME: ______________________ CLASS: __________ DATE: ________

Solve each problem.

1. Joan has two packages that weigh a total of 4.8 pounds. One package weighs 1.95 pounds. How much does the other package weigh?

2. Juan needs $1.43 to purchase a model car. He now has $.83. How much more money does he need in order to buy the model car?

3. The average yearly rainfall for Lowell County is 32.06 inches. Last year 29.75 inches of rain were recorded. How many inches below average was this?

4. Two sheets of plastic have a combined thickness of 1.08 inches. One sheet is .675 inch thick. What is the thickness of the other sheet?

5. Consider the numbers named by 4.32; 432; 43.2; and .432. What is the difference between the two greatest numbers?

6. Jane gave the clerk a $20 bill. The amount of her purchases was $14.32. How much change should she receive?

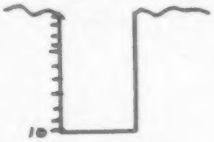
7.13.02 WORKING BACKWARDS

(1-2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.13.02.01A	make a model/chart of data in a problem; e.g., A frog fell in a hole 10 meters deep. Each day it was able to climb up 3 meters. Each night it slid back 2 meters. How many days did it take the frog to climb out of the hole?



*7.13.02.02A	use a model/chart to solve a problem by working backwards. e.g., Let us assume it took the frog 10 days to climb out of the hole. Use the model to prove or disprove your assumption.

GETTING STARTED

Write the following problem on the chalkboard.

> Three weeks ago six of my friends and I built a five foot high fence around my vegetable garden. The fence has seven posts on each side and is in the shape of a square. How many posts did we use in all?

Say, "Let's list all of the numerical information in this problem." Write the following on the chalkboard.

3 weeks
6 friends
5 feet high
7 posts

Say, "Which pieces of information are relevant and which are irrelevant to the solution of the problem?" List each under its proper heading.

e.g.,

Relevant	Irrelevant
seven posts	3 weeks
	6 friends
	5 feet high

Read the following problem to the class.

> A Boeing 747 jetliner, carrying 295 people, flew at 600 miles per hour between New York and Chicago. If the two cities are 900 miles apart by air, how long was the flight?

Elicit from the students the relevant data and list it on the chalkboard.

e.g.,

600 miles per hour
900 miles distance

Repeat this procedure using other problems.

7.13.02

7.13.02.01 **make a model/chart of data in a problem**

STRATEGY 1: Write the following problem on the chalkboard.

> How many different pairs of whole numbers can be added to total 40?

Say, "Can we use a chart to model the information in this problem? (Yes) What information must the chart show?" (Each possible pair and its sum)

Draw the following chart on the chalkboard.

e.g.,

Number Pair	Sum

Ask a student for a possible number pair (e.g., 34 and 6) and write it, along with its sum, on the chart. Say that by continuing to list all pairs, we would solve the problem.

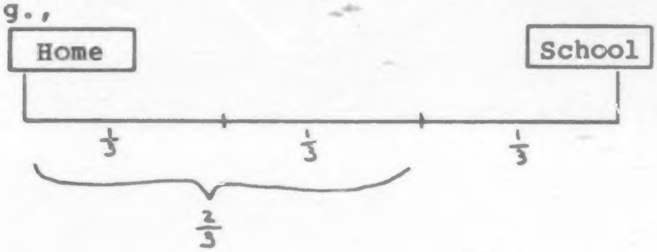
STRATEGY 2: Write the following problem on the chalkboard.

> Each day when I walk to school, I stop after ten blocks for a rest. At this point, I have completed two-thirds of the trip. How far is it from my house to school?

Say that the answer is not immediately apparent, and that a model of the problem will probably help us to see the solution.

Ask the students to devise a model using the information given in the problem.

e.g.,



7.13.02

7.13.02.02 use a model/chart to solve a problem by working backwards

STRATEGY 1: Draw the following chart on the chalkboard.

e.g.,

Start	4	6	10	13	
Finish	11	15	23		35

Say, "For the first three examples, both the start and finish numbers are given. Does someone see a pattern by which the start number became the finish number? (multiply by 2 and add 3) Following the same pattern, what will be the finish number for 13? (29) We can also work backward from the finish number to obtain the start number. What is the opposite of adding 3? (subtracting 3) What is the opposite of multiplying by 2? (dividing by 2) Working backward, step by step, what will be the start number for 35? (16) We can check this by reversing the path and using 16 as our starting number."

Repeat this procedure using different examples.

STRATEGY 2: Write the following problem on the chalkboard.

> Martin, Sandy and Alice each have some money. Martin gives Sandy two dollars and Alice one dollar. Then Sandy gives Martin three dollars and Alice one dollar. At the end of this process, they each have six dollars. With how much money did each of them start?

Say, "Because we know the final result (each one had six dollars), we can work backward, step by step, to find the situation at the beginning of the problem." (With how much money did each begin?)

STRATEGY 2: cont'd

Write on the chalkboard.

e.g.,

Martin	Sandy	Alice
6	6	6

Say, "What do these numbers represent? (the final amounts) What is the step just before this? (Sandy gives Martin three dollars and Alice one dollar.) How much money did Sandy give out? (four dollars) How much money must Sandy have had before giving out four dollars? (ten dollars) How much did Martin get? (three dollars) How much must Martin have had before getting three dollars? (three dollars) How much did Alice get? (one dollar) How much must she have had before getting one dollar?" (five dollars) Extend the chart as follows.

e.g.,

Martin	Sandy	Alice
6	6	6
3	10	5

Say, "Working backward, what was the previous step? (Martin gives Sandy two dollars and Alice one dollar.) How much did Martin give out? (three dollars) How much must he have had before this? (six dollars) How much did Sandy get? (two dollars) How much must Sandy have had just before this? (eight dollars) How much did Alice get? (one dollar) How much must she have had just before this?" (four dollars) Extend the chart as follows.

e.g.,

Martin	Sandy	Alice
$6	$6	$6
$3	$10	$5
$6	$8	$4

Elicit from the students that the bottom row of the chart (6,8,4) is the answer to the problem, and that it can be checked by reversing the process.

P.O. 7.13.02.01-.02

NAME:____ CLASS:__________ DATE:________

1. Draw a chart or table of the data in the problem.
2. Complete the chart, working backwards where necessary. Use the steps to help you.

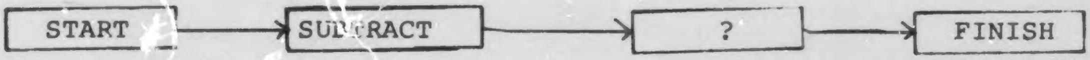
1. In my friend's backyard there are bicycles and tricycles. I counted 19 wheels. Make a chart of the possible number of bicycles and tricycles in the backyard.

2. When you roll two dice, what fraction of the possible products are single digit numbers?

3. If you toss three quarters, how many different ways can they possibly land?

4. How many odd numbers are there between 24 and 96?

5. Finish this chart.



START	8	15	6	12			
FINISH	21	42	15		60	45	81

7.14.02 FRACTIONAL RELATIONSHIPS - MIXED NUMBERS

(ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

7.14.02.01 change an improper fraction to a mixed number or a whole number;

e.g.,

$$\frac{19}{5} = 3\ \frac{4}{5}$$

$$\frac{15}{3} = 5$$

7.14.02.02 change a mixed number or a whole number into an improper fraction;

e.g.,

$$5\ \frac{2}{3} = \frac{17}{3}$$

$$6 = \frac{18}{3}$$

7.14.02.03 determine the least common denominator (LCD) of two or more fractions.

e.g.,

The least common denominator of $\frac{1}{3}$ and $\frac{1}{4}$ is $\frac{1}{12}$.

7.14.02

VOCABULARY

TERMS	SYMBOLS
mixed number	
whole number	
improper fraction	
least common denominator	LCD

GETTING STARTED

Write several sets, consisting of four proper fractions in unsimplified form, on the chalkboard.

e.g.,

$$\frac{4}{12}, \ \frac{2}{4}, \ \frac{6}{8}, \ \frac{4}{6}$$

Ask the students to simplify the fractions and rewrite each group in order from smallest to largest.

e.g.,

$$\frac{1}{3}, \ \frac{1}{2}, \ \frac{2}{3}, \ \frac{3}{4}$$

Repeat this procedure using other sets of fractions.

e.g..

$$\frac{3}{6}, \ \frac{2}{12}, \ \frac{3}{9}, \ \frac{3}{12}$$

$$\frac{3}{9}, \ \frac{2}{6}, \ \frac{2}{12}, \ \frac{4}{8}$$

List several pairs of fractions on the chalkboard.

e.g.,

$$\frac{2}{3} \ \square \ \frac{1}{4} \qquad \frac{9}{12} \ \square \ \frac{3}{4}$$

$$\frac{4}{8} \ \square \ \frac{1}{2} \qquad \frac{5}{9} \ \square \ \frac{11}{18}$$

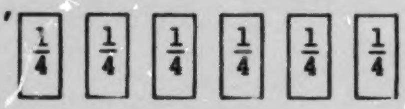
Instruct students to determine whether the first fraction is greater than, less than or equal to the second fraction. Ask the students to copy the pairs of fractions into their notebooks and insert the correct symbol between the two fractions. Ask several students to state their answers.

7.14.02

7.14.02.01 **change an improper fraction to a mixed number or a whole number**

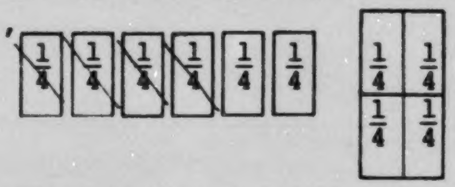
STRATEGY 1: Draw a diagram of a whole divided into fourths on the chalkboard. Say, "What is each part of this whole called?" Label each part with the fraction $\frac{1}{4}$. Say, "How many fourths are needed to make one whole?" Draw six separate parts at the chalkboard and label each part $\frac{1}{4}$.

e.g.,



Elicit from students the number of fourths drawn. Say, "Let's see how many wholes we can make from these fourths." Ask a student to reinforce the idea that four-fourths are needed to make one whole. Group your fourths together at the chalkboard. Cross off the parts of the original diagram that you have just used.

e.g.,



Say, "Can we make another whole from the fourths that are left over? (Another whole cannot be made because there are not enough fourths.) Six-fourths is equal to the whole and two-fourths." Write the equivalency at the chalkboard.

e.g.,

$$\frac{6}{4} = 1\frac{2}{4}$$

Ask the students to compare the $\frac{2}{4}$ with respect to the whole. Say that the $\frac{2}{4}$ is equal to half of the whole. Write the simplified equivalency at the chalkboard.

7.14.02

STRATEGY 1:
cont'd

e.g.,

$$\frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$$

Say that another reliable approach to change an improper fraction to a mixed number or a whole number is to use division. Explain the steps and demonstrate on the chalkboard using the preceding fractions as an example. Say, "Divide the denominator into the numerator. The number of times it is divided evenly is the number of wholes. The remainder, if any, becomes the fraction. Simplify the fraction if necessary."

e.g.,

$$\begin{array}{r} 1 \quad \text{R.2} \\ 4\overline{)\,6} \end{array} \qquad 1 \text{ whole} + \frac{2}{4} = 1\frac{2}{4} \quad \text{Simplify} = 1\frac{1}{2}$$

Repeat this procedure using other examples of improper fractions to be changed to mixed numbers.

STRATEGY 2:

Write the following statement on the chalkboard.

e.g.,

Is it true that $\frac{10}{8} = 1\frac{1}{4}$?

Ask the students the difference between the two fractions. Say that to determine whether or not the statement is true, you must use the operation of division. Ask the students to divide the denominator into the numerator. Write the response on the chalkboard. Ask the students if there is a remainder. Say that the remainder is written in the form of a fraction. The remainder becomes the numerator whereas the denominator remains the same.

e.g.,

$$\frac{10}{8} = 1\frac{2}{8}$$

Say that the answer is not in simplest form. Ask the students to simplify.

7.14.02

STRATEGY 2: cont'd

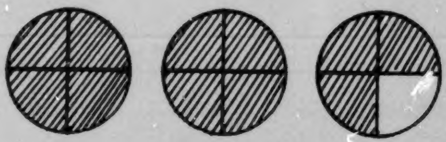
e.g., $\frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$

Ask a student to reread the statement on the chalkboard. Repeat this procedure using other examples of improper fractions and mixed numbers.

7.14.02.02 **change a mixed number or a whole number into an improper fraction**

STRATEGY 1: Say, "How can we determine how many fourths are in $2\frac{3}{4}$?" (draw a diagram) Then write the mixed number $2\frac{3}{4}$ on the chalkboard and draw three circles. Ask a student to divide each circle into fourths. Point to the first circle. Say, "How many fourths must I shade in on this circle?" Shade in all four sections using colored chalk. Repeat for the second circle. Point to third circle. Say, "How many fourths must I shade in to complete the fraction?" (3) Ask a student to count each of the shaded sections or fourths to see how many fourths are in $2\frac{3}{4}$.

e.g.,



Write $\frac{11}{4}$ on the chalkboard.

Say, "When you need to know how many fourths or fifths are in a larger mixed number like $9\frac{4}{5}$, the diagrams will take too long. There is a simpler way to solve the problem. First, we need to ask some questions. How many wholes are in $2\frac{3}{4}$? (2) What is the denominator in $2\frac{3}{4}$? (4) How many fourths

STRATEGY 1: cont'd

are in 2 wholes? (8) How many fourths are left?" (3) Write the following on the chalkboard.

e.g.,

2 wholes x 4 = 8

8 quarters + 3 quarters = 11 quarters

Write several examples on the chalkboard.
Change each mixed fraction to an improper fraction.

STRATEGY 2:

Say, "We need to determine how many thirds are in $4\frac{2}{3}$. First we find the number of thirds in four wholes, and then add the $\frac{2}{3}$.

e.g.,

$$4 \times \frac{3}{3} + \frac{2}{3} = \frac{14}{3}$$

This way, we have not changed the value of the fraction because $\frac{3}{3}$ equals 1 and we have simply multiplied by one."

Repeat this procedure for $2\frac{1}{6}$, $4\frac{1}{2}$, $5\frac{3}{12}$.

7.14.02.03 **determine the least common denominator (LCD) for two or more fractions**

STRATEGY 1:

Write the fraction $\frac{1}{3}$ and $\frac{1}{4}$ on the chalkboard. Say that we will be finding the least common denominator (LCD) for these two fractions. Ask for multiples of the three and four multiplication tables. Write these multiples next to the corresponding fraction.

e.g.,

$\frac{1}{3}$: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

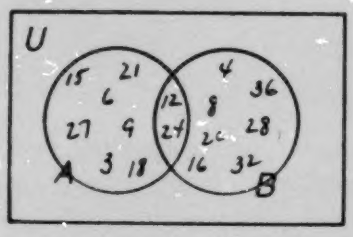
$\frac{1}{4}$: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

Ask a student to circle the numbers that are common

STRATEGY 1: cont'd

in both groups. Draw a Venn diagram on the chalkboard to represent both groups.

e.g.,



Say that Set A represents the multiples of three and Set B represents the multiples of four. Say that the area where the two sets intersect represents the multipliers that are common to both sets. Say, "Twelve is the least common denominator of $\frac{1}{3}$ and $\frac{1}{4}$ because it is smaller than twenty-four."

Repeat this procedure using other examples. Assist the students in observing that if there are two or more common multipliers, then the smallest number is the LCD.

STRATEGY 2:

Divide the class into two groups. Ask a student from the first group to write two fractions on the chalkboard, e.g., $\frac{1}{5}$, $\frac{1}{7}$. Ask a student from the second group to write the multiples of each denominator. Instruct the remaining students in the class to find the smallest number that is common to both sets. If the response is correct, that student has scored a point for his/her team.

e.g.,

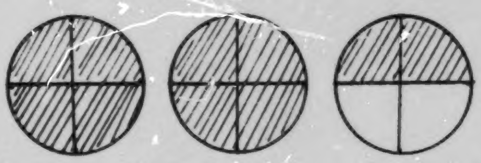
5 : 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
7 : 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
∴ LCD = 35

Repeat this procedure using other examples while the teams reverse responsibilities.

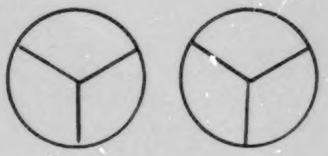
P.O. 7.14.02.01

NAME: ______________________ CLASS: __________ DATE: ________

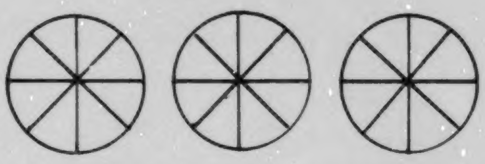
Shade each improper fraction. Change each improper fraction to a mixed number or a whole number. Simplify the fraction where necessary.



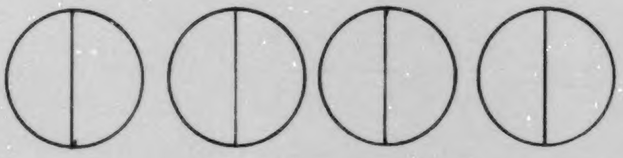
$\frac{10}{4} =$ $2\frac{2}{4} = 2\frac{1}{2}$



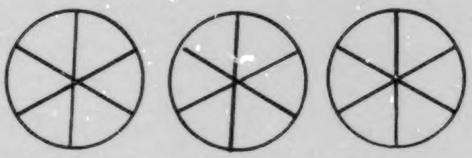
$\frac{5}{3} =$ ____________



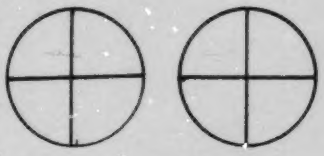
$\frac{10}{8} =$ ____________



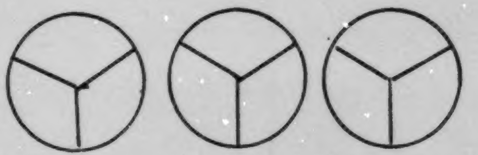
$\frac{8}{2} =$ ____________



$\frac{16}{6} =$ ____________



$\frac{7}{4} =$ ____________

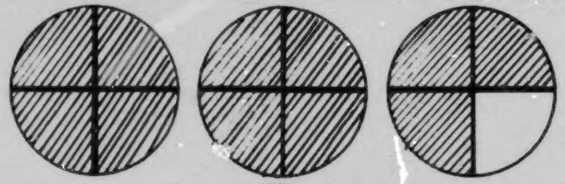


$\frac{6}{3} =$ ____________

P.O. 7.14.02.02

NAME: ______________________ CLASS: __________ DATE: ________

Write an improper fraction for each whole or mixed number. Show all work.



$2\frac{3}{4} = \frac{\boxed{11}}{4}$

$4 \times 2 = 8$

$8 + 3 = 11$

$6 = \frac{\square}{8}$	$9 = \frac{\square}{9}$	$16 = \frac{\square}{2}$	$11 = \frac{\square}{5}$
$4 = \frac{\square}{12}$	$5 = \frac{\square}{20}$	$6 = \frac{\square}{12}$	$7 = \frac{\square}{7}$
$4\frac{3}{4} = \frac{\square}{4}$	$5\frac{7}{8} = \frac{\square}{8}$	$8\frac{2}{5} = \frac{\square}{5}$	$9\frac{2}{3} = \frac{\square}{3}$

P.O. 7.14.02.03

NAME: ______________________ CLASS: __________ DATE: ________

Find the least common denominator for each group of fractions.

$\frac{1}{3}$ and $\frac{1}{5}$

$\{3,6,9,12,\underline{15},18,21,24,27,\underline{30}\}$

$\{5,10,\underline{15},20,25,\underline{30},35,40,45,50\}$

LCD = 15

$\frac{1}{8}$ and $\frac{1}{32}$

LCD =

$\frac{1}{12}$ and $\frac{1}{4}$

LCD =

$\frac{2}{5}$ and $\frac{3}{20}$

LCD =

$\frac{1}{8}$ and $\frac{1}{12}$

LCD =

$\frac{2}{3}$ and $\frac{4}{9}$

LCD =

$\frac{1}{4}$ and $\frac{1}{9}$

LCD =

$\frac{2}{9}$ and $\frac{4}{5}$

LCD =

$\frac{2}{7}$ and $\frac{1}{4}$

LCD =

$\frac{1}{7}$ and $\frac{1}{5}$

LCD =

$\frac{2}{5}$ and $\frac{3}{6}$

LCD =

$\frac{1}{8}$ and $\frac{2}{3}$

LCD =

Unit 20

7.06.01 ADVANCED FRACTION COMPUTATIONS AND CONCEPTS

(4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.06.01.01C find the average of two or more fractions; e.g.,

$$\frac{3}{4} + \frac{1}{2} = \frac{6}{8} + \frac{4}{8} = \frac{10}{8}$$

$$\frac{10}{8} \div 2 = \frac{\overset{5}{\cancel{10}}}{8} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{5}{8}$$

($\frac{5}{8}$ is the average of $\frac{3}{4}$ and $\frac{1}{2}$)

7.06.01.02C raise a fraction to an indicated power. e.g.,

$$(1\tfrac{1}{2})^2 = (1\tfrac{1}{2}) \times (1\tfrac{1}{2}) = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\tfrac{1}{4}$$

7.06.01

VOCABULARY

TERMS	SYMBOLS
average	
power	
exponent	
base	
parenthesis	( )
real number	

GETTING STARTED

Write several mixed numbers on the chalkboard, e.g., $4\frac{1}{3}$, $2\frac{1}{5}$. Ask the students what operations they can do with these mixed numbers. (They can be added, subtracted, multiplied or divided.) Ask four students to come to the chalkboard to each do one problem.

(1) addition

$$\begin{array}{r} 4\,\frac{1}{3} \\ +\;2\,\frac{1}{5} \\ \hline \end{array}$$

(2) subtraction

[illegible]

$$\begin{array}{r} \frac{1}{3} \\ -2\,\frac{1}{5} \\ \hline \end{array}$$

(3) multiplication

$$4\,\frac{1}{3} \times 2\,\frac{1}{5}$$

(4) division

$$4\,\frac{1}{3} \div 2\,\frac{1}{5}$$

Repeat this procedure using other examples.

Write several numbers with exponents on the chalkboard, e.g., 4^3. Ask the students what the base and the exponent represent. Say that an exponent tells how many times we multiply the base by itself, e.g., $4^3 = 4 \times 4 \times 4$. Write several examples on the chalkboard and ask the students to solve them, e.g., $(4.2)^3 = 4.2 \times 4.2 \times 4.2$. Say that any real number can be raised to a power. Present more examples for the students to solve.

7.06.01

7.06.01.01 **find the average of two or more fractions**

STRATEGY 1: Ask the students what the word average means. (An average is a mean score that expresses the total of all scores as one number.) Ask the students to give you some examples of averages that they deal with daily, e.g., test scores, batting averages, daily temperatures. Remind the students that the method for finding averages is to add up all the data and divide by the number of items added. Write several examples on the chalkboard.

e.g.,

(1) 5 test scores: 90, 65, 85, 70, 80

$$5\overline{)390}^{\,78} \qquad \text{average} = 78$$

(2) temperatures at different times during the day: 60°, 74°, 70°, 68°

$$4\overline{)272}^{\,68} \qquad \text{average} = 68°$$

Repeat this procedure using other statistics.

Ask the students how to find an average if the data is in fractional form. (It should not make a difference because the principle is the same.) Write $2\frac{3}{4}$ and $4\frac{4}{5}$ and say to the students that the rules are the same as those dealing with whole numbers. First we add the fractions.

e.g.,

$$\begin{array}{rcl} 2\frac{3}{4} & = & \frac{15}{20} \\ +4\frac{4}{5} & = & \frac{16}{20} \\ \hline 6 & & \frac{31}{20} = 7\frac{11}{20} \end{array}$$

Then we divide the sum by 2, because we added 2 items, e.g., $7\frac{11}{20} \div 2 = 7\frac{11}{20} \div \frac{2}{1} = \frac{151}{20} \times \frac{1}{2} = \frac{151}{40} = 3\frac{31}{40}$. $3\frac{31}{40}$ is the average of $2\frac{3}{4}$ and $4\frac{3}{4}$.

Repeat this procedure with more than two pieces of data. Ask students to come to the chalkboard to solve several averaging examples.

STRATEGY 2: Say, "How many people are in your immediate family?" Collect the data on the chalkboard, e.g., 4, 3, 4, 5, 8, 2, 4, 7... Say, "If you had to give a number to represent the number of people per family in this class, what would the number be called? (an average) How can we find the average of this data?" (We add the numbers and divide by the number of numbers.) Ask a student to find the average at the chalkboard.

Say, "What is the height of three boys in our class?" Record the answers in fraction form, e.g., $5\frac{1}{2}$ ft., $4\frac{3}{4}$ ft., ...

Ask the students if we can average this data. Elicit that data can be averaged whether it be fractions, whole numbers or decimals. Write the steps necessary to find an average on the chalkboard.

1) Add all the data.
2) Divide by the number of pieces of data that were added.
3) The resulting answer is the average.

Follow these steps to find the average height of the three boys.

e.g.,

$$\begin{array}{rr} 5\frac{1}{2} & \frac{9}{18} \\ 8\frac{2}{3} & \frac{12}{18} \\ +7\frac{4}{9} & \frac{8}{18} \\ \hline \multicolumn{2}{l}{20 + 1\frac{11}{18} = 21\frac{11}{18}} \end{array}$$

Say that the answer now has to be divided by 3 since there are 3 pieces of data that we added. Demonstrate that $21\frac{11}{18} \div 3 = \frac{389}{54} \times \frac{1}{3} = \frac{389}{162} = 2\frac{65}{162}$. $2\frac{65}{162}$ is the average height.

Repeat this procedure using other examples.

7.06.01

7.06.01.02 raise a fraction to an indicated power

STRATEGY 1: Write a base and exponent on the chalkboard, e.g., 2^3. Say that the base is "2", and the exponent tells how many times we multiply the base (2) by itself. Write a fraction in parenthesis on the chalkboard with an exponent, e.g., $(\frac{1}{3})^2$. Ask the students the meaning of this expression. Say that when anything is listed inside parenthesis, it is treated as one term. Demonstrate that $(\frac{1}{3})^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ and that $(2\frac{1}{3})^2 = 2\frac{1}{3} \times 2\frac{1}{3} = \frac{7}{3} \times \frac{7}{3} = \frac{49}{9} = 5\frac{4}{9}$. Repeat this procedure using other examples.

STRATEGY 2: Write a whole number on the chalkboard, e.g., 3. Ask the students if they can rename this number as a fraction, e.g., $\frac{3}{1}$. Say that any whole number can be expressed as an improper fraction. Ask the students if they think that 3 x 3 means the same as $\frac{3}{1} \times \frac{3}{1}$ or $\frac{6}{2} \times \frac{6}{2}$ or $\frac{3}{1} \times \frac{6}{2}$? Ask them to experiment to see that anything that 3 can do, $\frac{6}{2}$ or $\frac{3}{1}$ or $\frac{9}{3}$ will have the same result.

Write 3^4 on the chalkboard. Ask the students what this expression means. Elicit that 3^4 means 3 x 3 x 3 x 3. Ask the students if $(\frac{6}{2})^4$ means the same thing. Elicit that 3 x 3 x 3 x 3 = 81 and that $\frac{6}{2} \times \frac{6}{2} \times \frac{6}{2} \times \frac{6}{2} = \frac{1296}{16} = 81$. Say that a fraction can also be raised to any power. Repeat this procedure using the following examples.

Ex. 1) $(\frac{1}{2})^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Ex. 2) $(\frac{3}{2})^2 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$

Ex. 3) $(2\frac{1}{3})^2 = (\frac{7}{3})^2 = \frac{7}{3} \times \frac{7}{3} = \frac{49}{9} = 5\frac{4}{9}$

P.O. 7.06.01.01

NAME:____CLASS:_________DATE:________

Find the average of each pair of numbers. Match it to the answers on the right hand side of the page.

1. Sample $8\frac{1}{4} + 5\frac{15}{16} =$ $8\frac{8}{32} + 5\frac{30}{32} = 13\frac{38}{32} = 14\frac{6}{32} \div 2 = 7\frac{3}{32}$	$3\frac{5}{24}$
2. $9\frac{7}{12} + 1\frac{3}{4} =$	$7\frac{3}{32}$
3. $2\frac{2}{9} + 7\frac{5}{6} =$	$3\frac{13}{40}$
4. $3\frac{2}{3} + 2\frac{3}{4} =$	$5\frac{1}{36}$
5. $2\frac{1}{4} + 4\frac{2}{5} =$	$5\frac{2}{3}$

P.O. 7.06.01.02

NAME: ______________________ CLASS: ________ DATE: ________

Raise each base to its indicated power.

1. Sample

$6^3 = 6 \times 6 \times 6 = 216$

2. $\left(\frac{2}{3}\right)^3 =$

3. $\left(\frac{1}{6}\right)^2 =$

4. $\left(\frac{7}{4}\right)^2 =$

5. $\left(1\frac{2}{3}\right)^3 =$

6. $8^4 =$

7. $\left(4\frac{1}{2}\right)^2 =$

8. $2^3 =$

9. $\left(3\frac{1}{3}\right)^3 =$

10. $\left(5\frac{1}{8}\right)^2 =$

Unit 21

7.06.02 DIVISION WITH FRACTIONS

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

7.06.02.01A compute the quotient of two fractions using the algorithm: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$

e.g.,

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{5}{12}$$

7.06.02.02A solve verbal problems related to skills learned in this module

e.g.,

A carpenter wants to cut a 21 foot length of wood into $1\frac{1}{2}$ foot lengths. How many pieces will result?

$$21 \div 1\frac{1}{2} =$$

$$\frac{21}{1} \div \frac{3}{2} =$$

$$\frac{\overset{7}{\cancel{21}}}{1} \times \frac{2}{\underset{1}{\cancel{3}}} = 14 \text{ pieces}$$

7.06.02

VOCABULARY

TERMS

reciprocal
inverse

GETTING STARTED

Write the following problem on the chalkboard.
e.g.,

$$\frac{2}{3} \times \square = 1$$

Ask a student to fill in the missing factor $(\frac{3}{2})$.

Repeat this procedure with other examples which include whole numbers and mixed numbers.
e.g.,

$$\square \times 6 = 1$$
$$\frac{7}{9} \times \square = 1$$
$$\frac{8}{5} \times \square = 1$$
$$2\frac{1}{3} \times \square = 1$$

Ask the students to find the relationship between the two fractions. (The missing factor in each example is the inverse or reciprocal of the given factor.)

Write on the chalkboard.
e.g.,

$$8 \div 2 = \square \qquad 8 \times \frac{1}{2} = \square$$

Ask a student to find the answer to each problem. Say that dividing by 2 and multiplying by $\frac{1}{2}$ have the same result and that 2 and $\frac{1}{2}$ are called reciprocals. Ask the students to name the reciprocals of other numbers, e.g., 5, $\frac{2}{3}$. Say, "What is the result of multiplying a number by its reciprocal?" (1)

Repeat this procedure using other numbers, e.g., 9, $\frac{4}{5}$, 12, $\frac{7}{8}$.

7.06.02

7.06.02.01 **compute the quotient of fractions using the algorithm: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$**

STRATEGY 1: Write on the chalkboard, "Sue wants to walk from her home to school, which is a distance of $\frac{1}{2}$ mile. Each block on her way to school is $\frac{1}{10}$ mile long. How many blocks will she have to walk?"

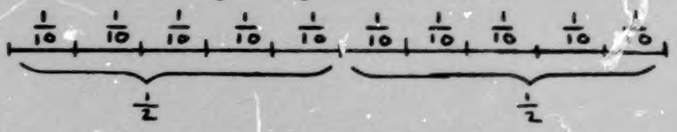
Say that we have to find how many $\frac{1}{10}$'s there are are in $\frac{1}{2}$, and we do this by dividing $\frac{1}{2}$ by $\frac{1}{10}$.

e.g.,

$$\frac{1}{2} \div \frac{1}{10}$$

Say, "How many $\frac{1}{10}$'s are there in one whole? (10) How many $\frac{1}{2}$'s are there in one whole?" (2)

Draw the following diagram on the chalkboard.



Ask the students to use the diagram to find out how many tenths there are in $\frac{1}{2}$. (There are five tenths in $\frac{1}{2}$.) Say that you can get the same answer by multiplying by the reciprocal.

Write the problem on the chalkboard.

e.g.,

$$\begin{aligned} \frac{1}{2} \div \frac{1}{10} &= \frac{1}{2} \times \frac{10}{1} \text{ (reciprocal)} \\ &= \frac{1}{2} \times \frac{10}{1} \\ &= 5 \end{aligned}$$

Repeat this procedure using other problems.

e.g.,

1) $\frac{1}{3} \div \frac{1}{6} =$

2) $\frac{1}{2} \div \frac{1}{4} =$

7.06.02

STRATEGY 1: cont'd

Ask a student to solve examples using mixed numbers.

e.g.,

$$10 \div 2\frac{1}{2} = \frac{10}{1} \div \frac{5}{2} \quad \text{(rename as a fraction)}$$

$$= \frac{10}{1} \times \frac{2}{5} \quad \text{(reciprocal)}$$

$$= \frac{{}^{2}\cancel{10}}{1} \times \frac{2}{\cancel{5}_{1}}$$

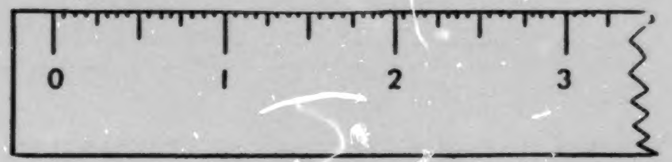
$$= 4$$

Repeat this procedure using other examples.

STRATEGY 2:

Write on the chalkboard, "How many $\frac{1}{16}$'s of an inch are there in $\frac{3}{4}$ of an inch?" Ask the students to use their rulers to determine the answer.

Draw a model of a ruler on the chalkboard or prepare an overhead transparency.



Ask a student to go to the model and demonstrate how to find how many $\frac{1}{16}$'s are in $\frac{3}{4}$ of an inch.

Write the numerical representation of the problem on the chalkboard.

e.g.,

$$\frac{3}{4} \div \frac{1}{16} = \frac{3}{4} \times \frac{16}{1}$$

$$= \frac{3}{{}_{1}\cancel{4}} \times \frac{\cancel{16}\,4}{1}$$

$$= \frac{12}{1} = 12$$

Repeat this procedure using other problems.

7.06.02

7.06.02.02 solve verbal problems related to skills learned in this module

STRATEGY 1: Prepare and distribute rexographed sheets with the following problem.

> A farmer who lives on a square plot of land decides to retire. He wants to keep one-fourth of the land for himself and give the rest to his four children. How may the land be divided so that the four children will have portions which are the same size?

Say, "What part of the land is left to be divided?" ($\frac{3}{4}$) Ask the students to translate the problem into a numerical expression.
e.g.,

$$\frac{3}{4} \div 4 =$$

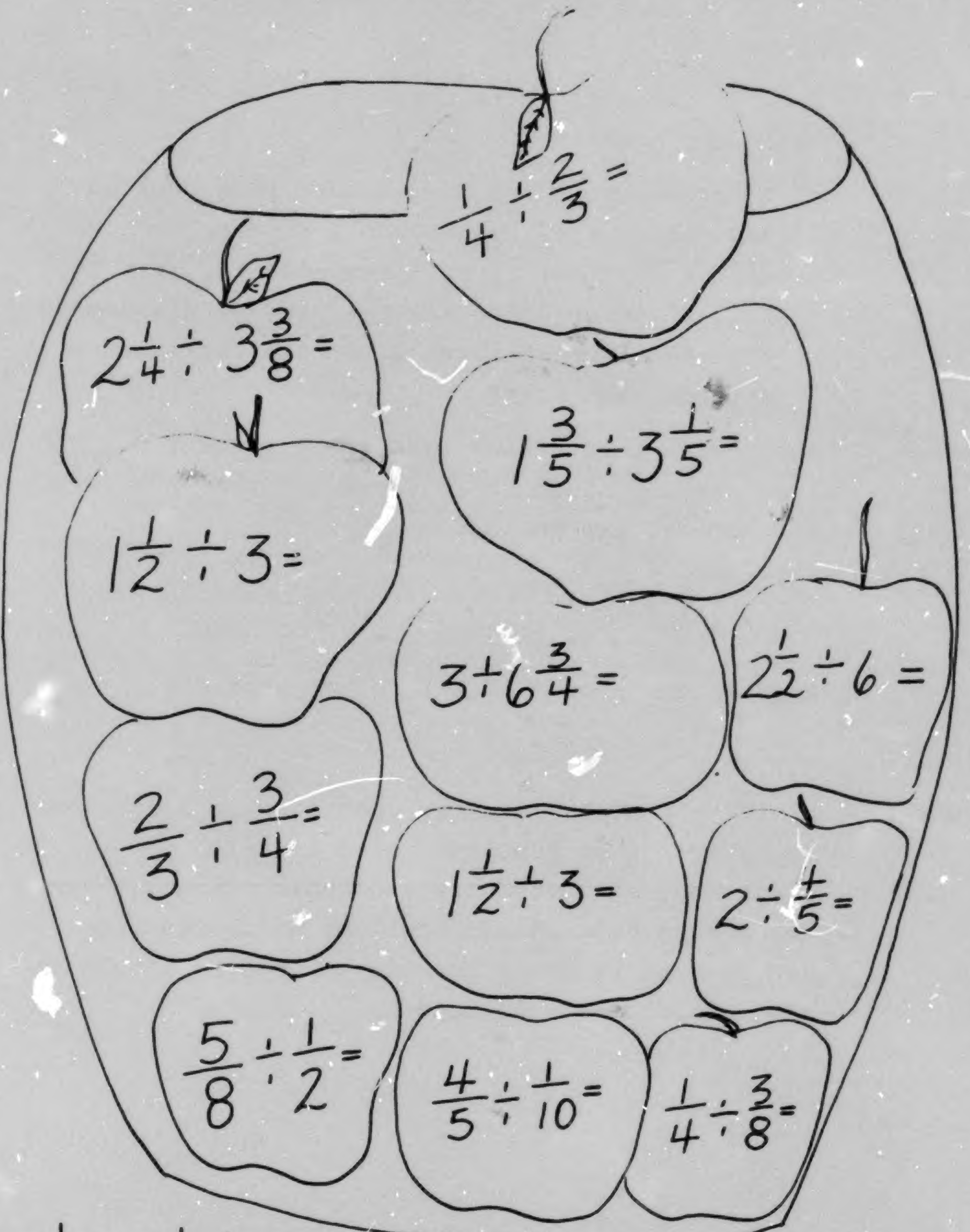
Repeat this procedure with other problems.

STRATEGY 2: Divide the class into two groups. Ask each group to make up five problems using division of fractions, mixed numbers and whole numbers. Ask the groups to exchange problems and solve them. Ask several students to come to the chalkboard to demonstrate their solutions.

P.O. 7.06.02.01

NAME: ______________________ CLASS: ________ DATE: ________

Write each quotient in simplest form.



$\frac{1}{4} \div \frac{2}{3} =$

$2\frac{1}{4} \div 3\frac{3}{8} =$

$1\frac{3}{5} \div 3\frac{1}{5} =$

$1\frac{1}{2} \div 3 =$

$3 \div 6\frac{3}{4} =$

$2\frac{1}{2} \div 6 =$

$\frac{2}{3} \div \frac{3}{4} =$

$1\frac{1}{2} \div 3 =$

$2 \div \frac{1}{5} =$

$\frac{5}{8} \div \frac{1}{2} =$

$\frac{4}{5} \div \frac{1}{10} =$

$\frac{1}{4} \div \frac{3}{8} =$

Remember what they say about "apples at the bottom of the barrel". Get to the top fast!

P.O. 7.06.02.02

NAME: ______________________ CLASS: __________ DATE: ________

Solve each problem. Write the answers in simplest form.

1. How many $\frac{1}{10}$ yard pieces of string can be cut from $\frac{4}{5}$ yard of string?

2. How many portions of pie can be served from 3 whole pies, if each portion was to be $\frac{1}{6}$ of a pie?

3. John can run a mile in $6\frac{1}{2}$ minutes. How many miles can he run in $12\frac{1}{2}$ minutes?

4. How many $2\frac{1}{2}$ pound packages of sugar can be filled from a $12\frac{1}{4}$ pound package?

5. How many quarts are there in $3\frac{1}{2}$ gallons? (Hint: 1 quart = $\frac{1}{4}$ gallon.)

7.06.03 RATIO AND PROPORTION

(5-6 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*7.06.03.01A compare two numbers by naming the ratio of one to another;
e.g.,

$$3 \text{ to } 4 \qquad 3:4 \qquad \frac{3}{4}$$

7.06.03.02A write equivalent ratios for a given ratio;
e.g.,

$$\frac{1}{4} \text{ is equivalent to } \frac{4}{16}$$

*7.06.03.03A identify a proportion as a statement of equality between two ratios;
e.g.,

$$\frac{3}{5} = \frac{6}{10} \quad \text{is a proportion}$$

$$3 \div 5 = 6 \div 10 \quad \text{is a proportion}$$

*7.06.03.04A identify means and extremes in a proportion;
e.g.,

$$\underbrace{3 \div \underbrace{5 = 6}_{\text{means}} \div 10}_{\text{extremes}}$$

7.06.03.05A determine if two ratios form a true proportion;

e.g.,

$$\frac{1}{2} \overset{?}{=} \frac{2}{4}$$

$$1 \times 4 \overset{?}{=} 2 \times 2$$

$$4 \overset{?}{=} 4$$

As $4 = 4$, we have a true proportion.

7.06.03.06A solve verbal problems involving related to learned in this module.

e.g.,

A floor pattern has 5 red tiles for every 6 blue ones. How many red tiles do I need if I have 60 blue ones?

$$\frac{5}{6} = \frac{n}{60}$$

$$5 \times 60 = n \times 6$$

$$300 = n \times 6$$

$$\frac{300}{6} = n$$

$$50 = n$$

50 red tiles needed

7.06.03

VOCABULARY

TERMS

ratio
proportion
means
extremes
equivalent

SYMBOLS

$: \quad \div \quad \frac{x}{y}$

GETTING STARTED

Write on the chalkboard, $\frac{1}{2} = \frac{\square}{10}$. Ask a student what number can go into this box to make this a true statement. Elicit 5. Ask the student to come to the chalkboard and fill in the number "5". Repeat this procedure using other examples, e.g., $\frac{1}{3} = \frac{\square}{12}$, $\frac{2}{5} = \frac{\square}{15}$. Write on the chalkboard, $\frac{2}{3}$. Ask a student if he/she can think of another fraction that represents the same thing. Ask another student if he/she can think of a different fraction that represents the same thing. Elicit $\frac{4}{6}$, $\frac{8}{12}$, $\frac{6}{9}$... Ask several students to come to the chalkboard and write in the answers.

Repeat this procedure using other examples.

Write on the chalkboard, $\frac{9}{15} = \square$. Ask a student to express the above fraction in its simplest form. Ask another student how the answer was obtained. Elicit $\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$ and write it on the chalkboard.

Repeat this procedure using other examples.
e.g.,

$\frac{6}{8}$, $\frac{8}{12}$, $\frac{12}{36}$.

7.06.03

7.06.03.01 **compare two numbers by naming the ratio of one to the other**

STRATEGY 1: Write the following on the chalkboard.

Mr. Johnson is 40 years old. His son Larry is ten years old. How many times older is Mr. Johnson than his son Larry?

Say, "How can we solve the problem? ($40 \div 10 = 4$ or $\frac{40}{10} = 4$) The quotient is equivalent to $\frac{4}{1}$ or 4. Therefore, Mr. Johnson is 4 times as old as his son. What we are really doing is comparing the two numbers. Another way of stating this is: the ratio of Mr. Johnson's age to his son's age can be written as 4:1." Write 4:1 on the chalkboard. Say to the students that we use 4:1, $\frac{4}{1}$ or "4 to 1" to show a ratio. Say, "Find the ratio of a quarter to a nickel. (25 to 5 is equivalent to 5 to 1; $\frac{25}{5}$ is equivalent to $\frac{5}{1}$; 25:5 is equivalent to 5:1.)

Repeat using other sets of numbers.

STRATEGY 2: Write the following in set notation on the chalkboard.

Set A = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Set B = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Say, "How is set A related to set B?" Elicit the following statements from the students.

1) Sets A and B are equivalent. Each set contains exactly 10 members.
2) Set A represents the natural or counting numbers from 1 to 10. Set B represents the even numbers from 2 to 20.
3) Set A contains members or elements which are exactly $\frac{1}{2}$ as much as its corresponding members or elements in Set B.

Using observation #3, say that we can show the relationship between Set A and Set B in this way.

STRATEGY 2: cont'd

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}, \frac{10}{20}.$

Say, "What is the same about all of these fractions? (They all equal $\frac{1}{2}$.) Another way of stating this is that each fraction is in the ratio of 1 to 2. This can be written mathematically as 1:2, $\frac{1}{2}$ or 1 to 2.

Let us express 5 inches to 1 foot as a ratio." Say that ratios should be expressed in the same unit of measure. Say that since 1 foot = 12 inches, this ratio should be expressed as 5 to 12 or $\frac{5}{12}$ or 5:12.

Repeat using other sets of numbers, e.g., 4 to 8, 6 to 7, ...

7.06.03.02 **write equivalent ratios for a given ratio**

STRATEGY 1: Write 8 to 4 on the chalkboard . Ask the students to write an equivalent ratio for 8 to 4. Ask a student to come to the chalkboard and give an example of an equivalent ratio, e.g., 2 to 1 is an equivalent ratio or 16:8 or 32 to 16 or $\frac{24}{12}$.
Ask the students how to find an equivalent ratio. (We multiply each term by the same number or divide each term by the greatest common factor.)

e.g., $\frac{8 \times 2}{4 \times 2} = \frac{16}{8}$

Say, "What is another name for $\frac{2}{2}$?" ($\frac{2}{2}$ is the multiplicative identity 1.)

Write a ratio on the chalkboard, e.g., $\frac{8}{4}$, and ask the students to use the multiplicative identity to find equivalent ratios.

e.g., $\frac{8 \times 3}{4 \times 3} = \frac{24}{12}$ $\frac{8 \div 2}{4 \div 2} = \frac{4}{2}$ $\frac{8 \div 4}{4 \div 4} = \frac{2}{1}$

7.06.03

STRATEGY 2: Draw on the chalkboard the representation of 16 boys to 20 girls.

e.g.,

B B B B B B B B B B B B B B B B
G G G G G G G G G G G G G G G G G G G G

Say, "How can we show the relationship of this group in other ways?" Four boys to five girls can be shown in the following ways.

B B B B   B B B B   B B B B   B B B B
G G G G G   G G G G G   G G G G G   G G G G G

Say, "We can write the ratio of 16 to 20 as 8 to 10 or 4 to 5 because they are all equivalent." Repeat this procedure using another pictoral representation and expressing the relationship in equivalent terms.

7.06.03.03 **identify a proportion as a statement of equality between two ratios**

STRATEGY 1: Attach six white chips and eight black chips to the chalkboard or an oak tag chart. Ask a student to write the ratio $\frac{6}{8}$ on the chalkboard in three different ways.

e.g.,

6 to 8, 6:8, $\frac{6}{8}$

Say, "Each ratio is to be read 6 to 8. This means for every six white chips there are eight black chips." Ask the students if the ratio $\frac{6}{8}$ can be reduced. ($\frac{6}{8}$ can be reduced to $\frac{3}{4}$.) Say that the ratio $\frac{6}{8}$ (6:8, 6 to 8) can also be written $\frac{3}{4}$, 3:4, 3 to 4.

Write on the chalkboard, $\frac{6}{8} = \frac{3}{4}$. Say that we call this expression a proportion. Ask a student to come to the front of the classroom and change the chips on the chalkboard to three white and four

STRATEGY 1: cont'd

black. Repeat the procedure using five white chips and ten black chips. Elicit from the students that $\frac{5}{10} = \frac{1}{2}$.

Repeat this procedure using $\frac{4}{8}$, $\frac{10}{12}$, $\frac{6}{9}$, $\frac{6}{10}$.

STRATEGY 2:

Ask one student to count the number of boys in the class, and another, to count the number of girls. Write the numbers as a ratio on the chalkboard, e.g., 12:15 or 12 to 15 or $\frac{12}{15}$. Say that for every 12 boys in the class there are 15 girls, and that the ratio is to be read 12 to 15. Ask the students if the ratio $\frac{12}{15}$ can be reduced. ($\frac{12}{15} = \frac{4}{5}$) Say that we call this expression a proportion.

Repeat this procedure using various examples.
e.g.,

10 daisies to 12 roses
6 cups to 8 plates
3 apples to 9 peaches

7.06.03.04 **identify means and extremes in a proportion**

STRATEGY 1:

Ask the students what they think the word "extreme" means. (Extremes can be thought of as temperatures, or grades, e.g., a test mark of 0 or 100 are extremes and that extremes usually mean "at the ends of the scale" or "the largest possible and least possible".) Ask the students what the term "means" means. (When we speak about means we usually mean "average" or "in the middle.")

Write $\frac{1}{3} = \frac{5}{15}$ on the chalkboard. Say that the extremes are the numbers that are at the ends of the scale, or in this case, 1 and 15, and that the means are the numbers in the middle of the scale, or in this case, 3 and 5. Repeat this procedure using other examples, e.g., $\frac{2}{5} = \frac{8}{20}$, $\frac{1}{6} = \frac{4}{24}$ ...

7.06.03

STRATEGY 2: Write a simple proportion on the chalkboard, e.g., $\frac{1}{2} = \frac{3}{6}$. Rewrite it like this, 1:2 = 3:6. Say to the students that extremes are at the ends and the means are in the middle. In this case the extremes are 1 and 6, and the means are 2 and 3. Repeat this procedure using other examples.
e.g., 2:4 = 8:16, 6:7 = 18:21.

7.06.03.05 **determine if two ratios form a true proportion**

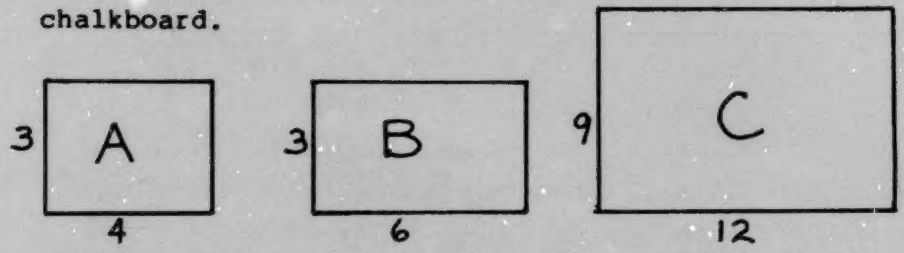
STRATEGY 1: Say, "What is a proportion? (A proportion is a statement formed by two equivalent ratios.) How can we be sure that two ratios are equivalent?" (We can express each ratio in simplest form and see if they are equal.) Ask the students to decide whether the following is a proportion by writing each ratio in the simplest form. Write on the chalkboard, 4:11 = 12:33. Say, "Name the means of the proportion. (The means are 11 and 12.) Name the extremes of the proportion." (The extremes are 4 and 3.) Write the following on the chalkboard.
e.g.,

```
4:11 = 12:33
↑  ↑    ↑  ↑
│  └means┘ │
└─extremes─┘
```

Say, "What is the product of the means? (11 x 12 = 132) What is the product of the extremes?" (4 x 33 = 132) Ask if 11 x 12 = 4 x 33.

Say, "When the product of the means equals the product of the extremes, the proportion is true. How do you think the product of the means will compare with the product of the extremes when the proportion is false? (The product will not be equal.) We call this the 'product test'." Repeat this procedure using 1:2 = 3:4, 9:12 = 3:4 and $\frac{6}{9} = \frac{3}{5}$.

STRATEGY 2: Prepare and display the following rectangles on the chalkboard.



Say, "Which rectangle do you think is similar to rectangle A? (Rectangle C is similar to rectangle A because they are in proportion.) What is the ratio of the width of A to the width of C? ($\frac{3}{9}$ or 3:9) What is the ratio of the length of A to the length of C? ($\frac{4}{12}$ or 4:12) How can we determine if these two ratios are equal?" (The product of the means should equal the product of the extremes.) Ask a student to come to the chalkboard to do the test, e.g., 3 x 12 = 4 x 9.

Say, "When the product of the means equals the product of the extremes, the proportion is true."

Repeat this procedure using 1:2 = 3:4, 5:7 = 25:35 and $\frac{6}{9} = \frac{3}{5}$.

7.06.03.06 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following on the chalkboard.

> The ratio of boys to girls in a class is 3:4. If there are 15 boys, how many girls are there in the class?

Say, "What proportion represents this problem?" (3 x ☐ = 4 x 15 and 3 x ☐ = 60 and ☐ = 20)

Repeat this procedure to solve other problems.

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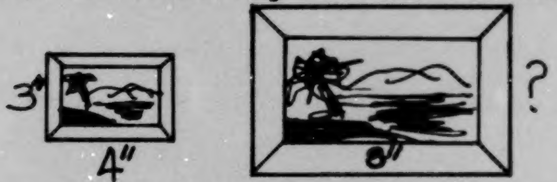
STRATEGY 2: Say, "Fred has a picture 3" wide and 4" long. He wants to enlarge it so that it will be 8" long. What will the width be?"

Draw the following on the chalkboard.



Say, "What is the ratio of the length of the original picture to the length of the enlarged picture? ($\frac{4}{8}$, or simplified, $\frac{1}{2}$ and therefore, the enlarged length is twice the original length.) To maintain the correct proportions, what should the ratio of the original width to the enlarged width be? (The ratio should also be $\frac{1}{2}$, the same as for the lengths.) Write a proportion that states that the ratios of the widths is the same as for the lengths." ($\frac{1}{2} = \frac{3}{?}$) Ask the students to find the unknown term. (1 x $\square$ = 2 x 3 and $\square$ = 6, so the enlarged length is 6)

Solve other problems using this procedure.

P.O. 7.06.03.01

NAME:______________________________ CLASS:_________ DATE:________

Match each problem in Column A with its correct ratio in Column B.

A		B
1. 15 to 3	________	a. 4:1
2. 14 to 35	________	b. 15:1
3. $\frac{30}{2}$	________	c. 3:4
4. $\frac{10}{30}$	________	d. 2:5
5. 10 oz. to 1 lb.	________	e. 6:1
6. 1m to 25 cm	________	f. 3:2
7. \$3 to 50¢	________	g. 3:1
8. 3½ hrs. to 30 min.	________	h. 7:1
9. 9 to 6	________	i. 5:1
10. 18 to 24	________	j. 2:9
11. 10 sec. to $\frac{3}{4}$ min.	________	k. 1:3
12. $\frac{3}{8}$ to $\frac{1}{8}$	________	l. 5:8

P.O. 7.06.03.02

NAME:________________________CLASS:________DATE:________

Match each of the following ratios in column I with an equivalent ratio in column II.

Column I		Column II
1. 5 to 6	______	a. 10 to 15
2. 8:9	______	b. 1:2
3. 1 to 7	______	c. 56:63
4. $\frac{3}{8}$	______	d. $\frac{4}{1}$
5. $\frac{2}{3}$	______	e. 6 to 8
6. 15:75	______	f. $\frac{3}{21}$
7. 13 to 26	______	g. 20 to 24
8. 7 to 11	______	h. $\frac{27}{72}$
9. 12:3	______	i. 3:15
10. $\frac{3}{4}$	______	j. 14:22

P.O. 7.06.03.03

NAME: ______________________ CLASS: __________ DATE: ________

Fill in each box to make a true proportion.

1. $\frac{8}{\square} = \frac{6}{12}$

2. $\frac{7}{14} = \frac{\square}{16}$

3. $\frac{7}{35} = \frac{1}{\square}$

4. $\frac{6}{36} = \frac{\square}{6}$

5. $\frac{\square}{15} = \frac{3}{5}$

6. $\frac{3}{4} = \frac{\square}{12}$

7. $\frac{\square}{10} = \frac{1}{2}$

8. $\frac{16}{20} = \frac{\square}{5}$

9. $\frac{9}{15} = \frac{\square}{5}$

10. $\frac{18}{27} = \frac{\square}{3}$

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P.O. 7.06.03.04

NAME: ______________________ CLASS: ________ DATE: ________

Identify the means and extremes in the following proportions.

1. $\frac{2}{5} = \frac{5}{30}$

means 5 , 5

extremes 2 , 30

2. $\frac{1}{6} = \frac{5}{30}$

means ____, ____

extremes ____, ____

3. $\frac{3}{4} = \frac{12}{16}$

means ____, ____

extremes ____, ____

4. $\frac{1}{9} = \frac{3}{27}$

means ____, ____

extremes ____, ____

5. 3 : 4 = 6 : 8

means ____, ____

extremes ____, ____

6. 4 : 5 = 8 : 10

means ____, ____

extremes ____, ____

7. 6 : 7 = 18 : 21

means ____, ____

extremes ____, ____

8. 11 : 12 = 33 : 36

means ____, ____

extremes ____, ____

P.O. 7.06.03.05

NAME: ______________________ CLASS: __________ DATE: ________

A. Is the proportion true?

1. $\frac{3}{4} \overset{?}{=} \frac{24}{32}$

   3x32= 24x4
   96 = 96

   Yes

2. $\frac{3}{4} \overset{?}{=} \frac{7}{9}$

3. $\frac{3}{4} \overset{?}{=} \frac{8}{12}$

4. $\frac{6}{4} \overset{?}{=} \frac{3}{2}$

5. $\frac{8}{3} \overset{?}{=} \frac{24}{9}$

6. $\frac{5}{9} \overset{?}{=} \frac{30}{56}$

7. $\frac{5}{9} \overset{?}{=} \frac{30}{56}$

8. $\frac{33}{11} \overset{?}{=} \frac{9}{3}$

9. $\frac{2\frac{1}{2}}{8} \overset{?}{=} \frac{5}{16}$

10. $\frac{0.4}{0.5} \overset{?}{=} \frac{1.6}{2}$

B. Use the following set of numbers to form a true proportion.

(1) 5, 15, 30, 10

(2) 6, 8, 4, 12

(3) 1, 4, 10, 40

(4) 4, 22, 55, 10

P.O. 7.06.03.06

NAME: ______________________ CLASS: ________ DATE: ________

Solve each problem.

1. The scale on a map is 1 inch = 500 miles. If two cities are 1500 miles apart, how far apart are they on the map?

2. Maria works in the local supermarket after school. She makes a minimum wage of $3.35 per hour. At the end of the week she clocked 8 hours. What was her gross pay?

3. Robert traveled 11 miles on his bicycle in 2 hours. At the same rate, how long will it take him to travel 44 miles?

4. David scores an average of 7 foul shots out of every 10 attempts. At the same rate, how many shots would he score in 200 attempts?

5. The scale on a blueprint is 1 inch = 20 feet. The length of the room on the blueprint is 1¼ inches, what is the actual length of the room?

Unit 23

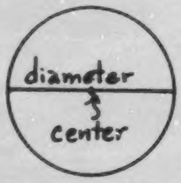
7.09.04 CIRCLES

(5 lessons)

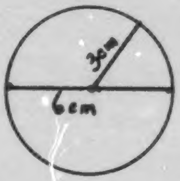
PERFORMANCE OBJECTIVES

The student will be able to...

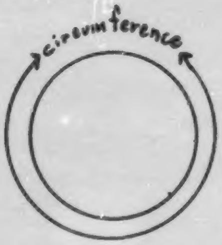
*7.09.04.01A identify a diameter as the longest chord passing through the center of a circle;



*7.09.04.02A use the relationship between the radius and the diameter (D = 2R);
e.g.,



*7.09.04.03A identify the circumference as the distance around the circle;
e.g.,



*7.09.04.04A state the definition of π as the ratio of the circumference of a circle to the diameter;

e.g.,

Diameter	Circumference Approximately
3	9.42
6	18.84

Circumference is a little more than 3 times the diameter ($\pi = \frac{C}{D}$)

$$\pi \simeq 3.1416$$

$$\pi \simeq \frac{22}{7}$$

*7.09.04.05A state the decimal and fractional approximations for π;

e.g.,

$$\pi \simeq 3.14$$

$$\pi \simeq \frac{22}{7}$$

7.09.04.06A use the formula for circumference of a circle where the radius ($C = 2\pi r$) or the diameter ($C = \pi d$) is given;

e.g.,

radius = 6 inches

$C \simeq (2)(3.14)(6)$ 37.68 inches

diameter 12 inches

$C \simeq (3.14)(12)$ 37.68 inches

7.09.04.07A solve verbal problems related to skills learned in this module.

e.g.,

The diameter of a wheel is $3\frac{1}{2}$ ft. How many feet will the wheel travel in one complete turn (1 revolution)?

$C \simeq (3.14)(3\frac{1}{2})$ 10.99 ft.

7.09.04

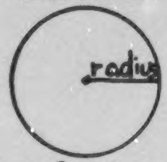
VOCABULARY

TERMS	SYMBOLS
radius	R/r
diameter	D/d
chord	
circumference	C
pi	π

GETTING STARTED

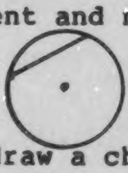
Draw a circle on the chalkboard and ask a student to locate and mark the center. Say, "How can you be sure that this point is the center?" (The point should be equidistant from all points on the circle.) Draw a line from the center to the circle. Ask a student to label this line (radius).

e.g.,



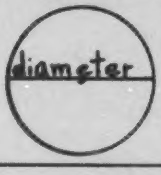
Ask a student to select any two points on the circle and connect them with a line segment and name the segment (chord).

e.g.,



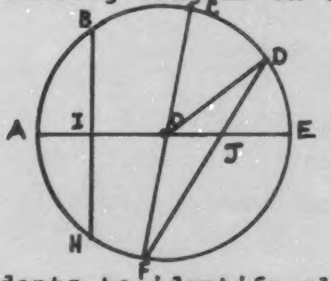
Ask students to draw a chord that passes through the center of the circle and name it the diameter.

e.g.,



Draw the following diagram on the chalkboard.

e.g.,



Ask the students to identify all radii, diameters and chords.

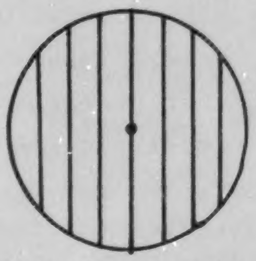
7.09.04

7.09.04.01 **identify a diameter as the longest chord passing through the center of a circle**

STRATEGY 1: Ask the students to draw a circle and locate its center in their notebooks. Ask the students to use a ruler to measure the length of the radius, and then the length of the diameter. Say that the diameter is a chord of the circle. Ask the students to draw a few other chords in their circles. Ask the students to measure each chord. Say, "How do the measures of the chords vary as the chords are closer to the center? (The closer the chords are to the center, the greater is the measure.) Which is the longest chord?" (The diameter, which passes through the center, is the longest chord)

STRATEGY 2: Draw the following diagram on the chalkboard.

e.g.,



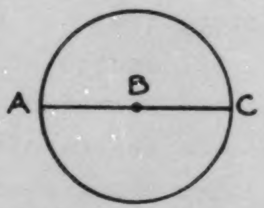
Say, "What do we call each of these lines? (chords) What is the name of the chord that passes through the center of the circle?" (diameter)

Ask the students to compare, visually, the lengths of the chords as we get closer to the center of the circle. (The closer that the chords are to the center, the greater is the measure of their length.) Say, "Which is the longest chord?" (the diameter)

7.09.04

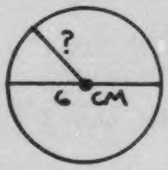
7.09.04.02 **use the relationship between the radius and the diameter (D = 2R)**

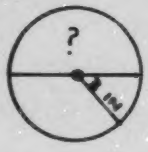
STRATEGY 1: Draw the following diagram on the chalkboard.
e.g.,

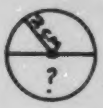


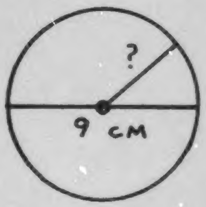
Say, "What do we call $\overline{AC}$? $\overline{AB}$? $\overline{BC}$? (diameter or radii) How many radii are contained in the diameter? (2 radii are contained in the diameter) Each radius is one half of the diameter. Symbolically, we can write $R = \frac{1}{2}D$ to express this relationship. Similarly, the relationship $D = 2R$ can be written to show that the diameter is twice the radius."

STRATEGY 2: Prepare and distribute a sheet which has three different circles on it. Ask the students to draw and measure a diameter on each circle and record the length under the circle. Repeat this procedure for the radii. Say, "Do you see a pattern between the length of the radius and the diameter of each circle?" (The radius is one half the diameter ($R = \frac{1}{2}D$) or the diameter is twice the radius ($D = 2R$).) Say, "Using these relationships, find the missing part in each circle."
e.g.,





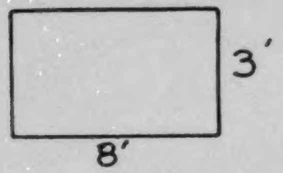




7.09.04

7.09.04.03 **identify the circumference as the distance around the circle**

STRATEGY 1: Draw the following rectangle on the chalkboard. e.g.,



Ask the students to find the distance around the geometric figure. (3+8+3+8 = 22) Say that the distance around a geometric figure is called its "perimeter", and we find the perimeter by adding the lengths of all sides. Say that we call the distance around a circle a circumference. Ask the students how they would find the length of a circumference. Say, "Would you use the same method as you would use to find a perimeter? What methods can be used?" (We can wrap a tape measure or a length of string, which will be later measured, around the circumference to find its length.) Say that another method for measuring the circumference is to rotate the circular object on a line or ruler for one full rotation and measuring the distance between the starting and ending points as diagrammed below. Ask the students to measure the circumference of various circular objects in the classroom.)

STRATEGY 2: Say that in Marine Park in Brooklyn there is a circular track that meaures one mile around. Say that we call the distance around a circle its circumference. Say, "What is the circumference of this circular track?" (one mile) Say that the

7.09.04

STRATEGY 2: cont'd

circumference of a circle is analogous to the perimeter of a geometric figure. Ask the students how they would find the length of a circumference. Elicit that we can wrap a tape measure or a length of string, which can later be measured, around the circumference to find its length.

7.09.04.04 state the definition of π as the ratio of the circumference a circle to the diameter

STRATEGY 1: Prepare and show a cup, a small clock and a 45 rpm record. Write on the chalkboard.

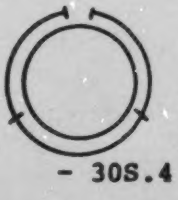
Item	Diameter	Circumference

Use a tape measure to measure the diameter of each item and then the circumference. Record the measures in the chart. Say, "Can you see any relationship between the diameter and the circumference?" Elicit that the circumference is a little more than three times the diameter. Say that we call this relationship π, and the value of π is $\frac{\text{circumference}}{\text{diameter}}$ or $\pi = \frac{C}{D}$.

Continue to measure circular objects and find the relationship $\frac{C}{D}$.

STRATEGY 2: Draw a circle on the chalkboard and use string, yarn or rope to measure the diameter. Cut the length exactly. Place the length of string around the circle, marking the starting and ending position.

e.g.,



7.09.04

STRATEGY 2: cont'd

Say that the yarn goes around the circle three times with a little left over. Ask several students to experiment with circle measurement using the yarn or string. Ask the students if they can see any relationship between the circumference and the diameter. (The circumference is three times and a little more than the diameter.) Say that we call this relationship pi (π) and it is found by dividing the circumference by the diameter of any circle.

7.09.04.05 **state the decimal and fractional approximations for π**

STRATEGY 1:

Ask the students to state the relationship of the circumference of a circle to the diameter. ($\frac{C}{D} = \pi$) Draw a circle and label the diameter 66 cm and the circumference 207.24 cm π. Say, 207.24 ÷ 66 = 3.14 and we call this π. Repeat this procedure using a circle with a diameter of 7 in. and a circumference of 22 in.

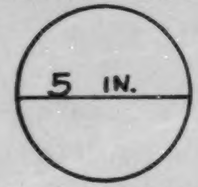
Say to the students that approximations for π are 3.14 or $3\frac{1}{7}$ which equals $\frac{22}{7}$.

7.09.04.06 **use the formula for circumference of a circle where the radius ($C = 2\pi r$) or diameter ($C = \pi d$) is given**

7.09.04

STRATEGY 1: Say that the relationship between the circumference and diameter is called pi (π). Say, "What is the value of pi (π)?" (3.14 or $\frac{22}{7}$) Draw a circle on the chalkboard and label the diameter 5 in.

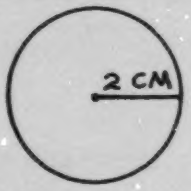
e.g.,



Say that the circumference is equal to π times the diameter. Write $C = \pi d$ on the chalkboard. Ask a student to find the circumference of the circle. ($C = \pi(5) = 3.14(5) = 15.7$ in.)

Draw a circle with a radius of 2 cm. on the chalkboard.

e.g.,



Ask the students how to find the circumference if we only know the length of the radius. (We double the length of the radius, to equal the diameter, and then multiply by pi (π), or $C = 2\pi r$) Ask a student to find the circumference of the circle on the chalkboard.

e.g., $C = 2\pi(2) = 2(3.14)(2) = 12.56$ cm.

Repeat this procedure using several circles where the diameter or radius has been given.

STRATEGY 2: Prepare and distribute circular objects, a ruler and string to each pair of students. Ask the students to measure the diameter with the ruler and the circumference with the string. Say to the students that the relationship between the circumference and the diameter is called pi (π) and $\pi = \frac{C}{D}$. Develop the formula $C = \pi D$ by multiplying

STRATEGY 2: cont'd

both sides of the equation by D, e.g., $D \times \pi = \frac{C}{D} \times D$. Ask the students to recall the value of π. ($\pi = 3.14$ or $\frac{22}{7}$) Ask the students to use the formula to find the circumference of the circular objects, e.g., $C = 3.14(D)$ or $C = \frac{22}{7}(D)$. Say, "Is the circumference the same if we use the formula or the string?" Ask students to change objects and continue to measure circumference using both the string and the formula.

7.09.04.07 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following problem on the chalkboard.

> Ralph has a bicycle with wheels 26 inches in diameter. How far does he travel in one turn of the wheel?

Say, "How can we find the solution to this problem?" (The distance traveled is the circumference of the wheel, and so given the diameter of the circle, we need to find the circumference.) Ask the students to name the procedures for problem solving.

e.g.,

1) Read the problem.
2) Select the appropriate operation.
3) Estimate the answer.
4) Solve the problem.
5) Check the answer.

Ask a student to come to the chalkboard and solve the problem using these steps.

e.g., $C = 3.14 \times 26 = 81.64$ inches

Repeat this procedure using other problems.

STRATEGY 2: Ask the students to name the steps in good problem solving.

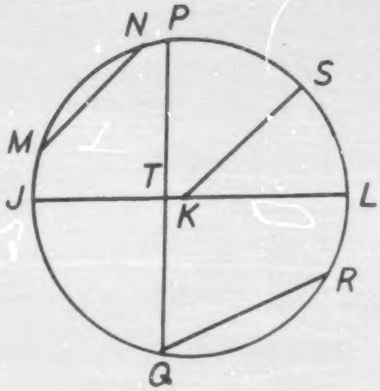
e.g.,

1) Read the problem.
2) Determine the operation using key words.
3) Estimate the answer by rounding.
4) Solve the problem.
5) Check your answer.

Write the steps on the chalkboard or a chart. Prepare and distribute index cards with a verbal problem on each. Ask the students to solve the problem on his/her card and exchange the problem with a neighbor. Ask several students to write the problems and solutions on the chalkboard.

P.O. 7.09.04.01

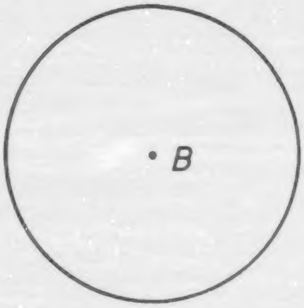
NAME:______________________ CLASS:________ DATE:________



The figure at the left is a circle with its center at K.

a) Identify all chords in the circle.
b) Which is the longest chord?

2. Draw the longest possible chord in the circle at the right which has its center at B. How many of these longest chords may be drawn?



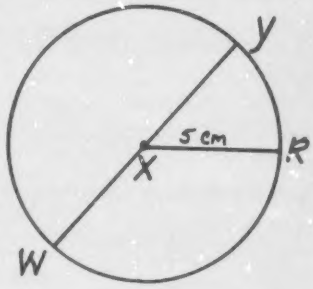
3. Two chords are drawn in a circle. One passes through the center, and the other does not pass through the center. Which is the shorter? Why?

229

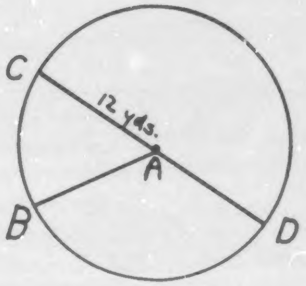
P.O. 7.09.04.02

NAME: ____________________ CLASS: __________ DATE: ________

1. In the circle drawn at right with center at X, find the measure of $\overline{WY}$ if $\overline{XR}$ = 5cm



2. In the circle drawn at right with center at A, find the measure of $\overline{BA}$ if $\overline{CD}$ = 12 yards



3. What is the length of the diameter of a circle whose radius measures:

a) 4" b) 29' c) 5.8 yds. d) 3½ cm

4. What is the length of the radius of a circle whose diameter measures:

a) 18' b) 62" c) 9 m d) 12.4 yds.

P.O. 7.09.04.03-.06

NAME:______________________________ CLASS:__________ DATE:________

Use $\pi \simeq 3.14$ to find the circumference of a circle whose diameter is:

1. 49 feet
2. 68.3 cm
3 4½ yds.

Use $\pi \simeq 3.14$ to find the circumference of a circle whose radius is:

4. 5 miles
5. 17.2 inches

Use $\pi = \frac{22}{7}$ to find the circumference of a circle whose diameter is:

6. 35 feet
7. 200 inches

Use $\pi = \frac{22}{7}$ to find the circumference of a circle whose radius is:

8. 140 cm
9. 18.2 yds.

P.O. 7.09.04.07

NAME: ______________________________ CLASS: __________ DATE: ________

Solve each problem.

1. The diameter of the wheels of Susan's 10 speed racer is 27". How many inches does Susan move in one turn of the wheels?

2. A circular man-made lake has a radius of 560 feet. How many feet of lake-front property can be located around the lake?

3. A belt is wrapped around a circular wheel in a machine. The wheel has a radius of 5.8 inches. How far will the belt move with one complete revolution of the wheel?

4. A circular plot of land which has a diameter of 350 meters is to be roped-off. How many feet of rope will be necessary?

Unit 24

7.10.01 MEASUREMENT - LINEAR (ENGLISH)

(2 lessons)

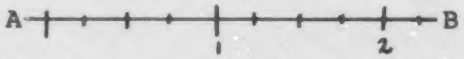
PERFORMANCE OBJECTIVES

The student will be able to...

7.10.01.01A select the appropriate unit for measuring length;
e.g.,

A fence around a garden (foot or yard);
The length of a football field (yard);
A homerun (foot or yard).

*7.10.01.02A measure a given line segment with a standard ruler in English units to the nearest 1 in., $\frac{1}{2}$ in., $\frac{1}{4}$ in., $\frac{1}{8}$ in.;
e.g.,



2 inches to the nearest inch;

$2\frac{1}{2}$ inches to the nearest $\frac{1}{2}$ inch;
$2\frac{3}{8}$ inches to the nearest $\frac{1}{8}$ inch.

7.10.01.03A convert a given measure of length to another equivalent unit of length;
e.g.,

1 yard = 36 inches
$\frac{2}{3}$ ft. = 8 inches

7.10.01.04A solve verbal problems related to skills learned in this module.

e.g.,

If 30 inches of leather will make one belt, how many feet of leather would be needed to make 12 belts?

12 x 30 inches = 360 inches
360 inches = 30 feet
30 feet of leather

7.10.01

VOCABULARY

TERMS

linear

GETTING STARTED

Prepare and distribute one whole straw, two $\frac{1}{2}$ straws and four $\frac{1}{4}$ straws to each student.

Ask the students to measure several objects using the straws, e.g., length of arm, foot, notebook, desk.

Illustrate one item to the class, e.g., "My arm measures one whole straw plus $\frac{1}{2}$ straw plus about $\frac{1}{4}$ straw. Altogether my arm measures approximately $1\frac{3}{4}$ straws."

Prepare and distribute an eight inch length of string, a ruler and a pair of scissors. Ask the students to cut several lengths from the string, e.g., a two inch piece, a half inch piece. Ask the students to measure several items using the string, e.g., a pencil, a shoe, your pinky, your thumb. Ask the students to estimate the length before actually measuring.

7.10.01

7.10.01.01 **select the appropriate unit for measuring length**

STRATEGY 1: Ask the class to name some standard units of length in the English system of measurement, our present system, e.g., inch, foot, yard, mile. Display a foot ruler, a yardstick, a tape measure, a carpenter's rule. Ask the students which unit of measurement would be appropriate and which tool they would use to measure each of the following items: length of a book, length of a classroom door, length of your shoe, length of school hallway, your height, thickness of a page in a book, diameter of a quarter, distance from New York to Miami.

STRATEGY 2: Ask the class to spread their fingers apart as far as they can. Say, "The distance from the tip of your thumb to the tip of your pinky is called your hand span. Measure the top of your desk in hand spans." List results on the chalkboard. Ask the class why it is that not everyone got the same answer. (All handspans are not the same.) Distribute a ruler to each student. Ask the class to measure their handspans to the nearest inch.

Display various measuring devices, e.g., tape measure, yardstick, trundle wheel. Ask which measuring tool could be used to measure an arm span. Ask a student to spread his/her arm span with the suggested tool.

Ask what devices could be used to measure a running race distance. (tape measure, trundle wheels)

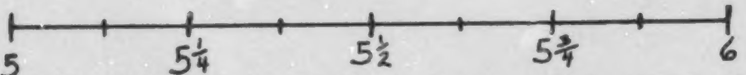
Repeat this procedure using other measuring tools.

7.10.01

7.10.01.02 **measure a given line segment with a standard ruler in English units to the nearest 1 in., $\frac{1}{2}$ in., $\frac{1}{4}$ in., $\frac{1}{8}$ in.**

STRATEGY 1: Prepare and distribute rulers divided into $\frac{1}{8}$ or $\frac{1}{16}$ inch units to each student. Draw a "magnified" inch on the chalkboard as illustrated.

e.g.,



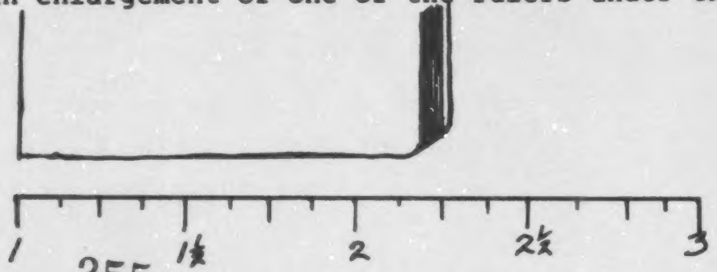
Ask the students to define the partitions of the inch. Say, "I am going to measure my pen to the nearest inch. The pen is not quite 5$\frac{1}{2}$" long, therefore it is closest to 5". When measuring objects against a ruler, we need to determine to which measurement the object is closest, e.g., 5$\frac{1}{2}$" is closer to 5" than 6", 5$\frac{3}{8}$" is closer to 5$\frac{1}{2}$" than 5". Now measure your pens or pencils to the nearest inch."

Record some of the responses on the chalkboard.

Repeat this procedure measuring objects to the nearest $\frac{1}{4}$ in., $\frac{1}{8}$ inch.

STRATEGY 2: Draw three magnified rulers on the chalkboard. Show 2 inches, 3 inches and 4 inches. Prepare three enlarged objects, e.g., a pencil, a book, a stapler. Remind the students that these are not true figures, but enlargements. Ask the students to identify halves, fourths and eighths of an inch. Place an enlargement of one of the rulers under the book.

e.g.,



7.10.01

STRATEGY 2: cont'd

Say, "How long is this book to the nearest inch? Why? (2 inches; it is closer to 2 than 1) To the nearest half inch? ($1\frac{1}{2}$ inch) To the nearest quarter of an inch? ($1\frac{3}{4}$ inch) To the nearest eighth of an inch?" ($1\frac{5}{8}$ inch) Repeat this procedure for the other enlarged objects. Prepare and distribute rulers to each student. Ask the students to measure several objects to the nearest $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ inch.

7.10.01.03 **convert a given measure of length to another equivalent unit of length**

STRATEGY 1: Develop this Table of Measurement with the students.

e.g.,

12 inches = 1 foot
36 inches = 1 yard
3 feet = 1 yard
5280 feet = 1 mile
1760 feet = 1 mile

Say, "We are going to play a measurement game. I will say a measurement, e.g., 3 feet. You will raise your hand when you can think of another way to name this measurement, e.g., 1 yard, 36 inches. If you are correct, you will earn a point." Play several rounds of the game. Ask a student to come to the chalkboard and show how to convert 27 feet into yards.

e.g.,

$$\frac{3 \text{ ft.}}{1 \text{ yd.}} = \frac{27 \text{ ft.}}{\square} \quad \text{or} \quad 3\overline{)27}$$

$$27 \times 1 = 3 \times \square$$

Ask several students to demonstrate other conversions using ratio, multiplication and division. Continue playing the measurement game.

STRATEGY 2: Say, "Which of these units would you use to measure the distance between Chicago and New York? Between your chair and my desk? The height of the school? The length of the chalkboard? Your height?" List the responses on the chalkboard.

Write this list on the chalkboard.

12 inches = 1 foot
3 feet = 1 yard
5280 feet = 1 mile

Say, "If 3 feet = 1 yard and 5280 feet = 1 mile, how many yards = 1 mile?" (Since fewer yards equal a mile than feet, division is used to find the answer.)

e.g.,

$$\frac{5280 \text{ feet}}{3 \text{ feet}} = 1760 \text{ yards (in one mile)}$$

Ask three students to measure the width of the classroom. Ask them to read their measurements, first in feet, then in inches, and write the results on the chalkboard. Have three different students measure the length of the chalkboard in feet and in inches. Ask them to write the measurements on the chalkboard.

Ask the class to convert all of these measurements into yards. Ask two other students to measure the length of the chalkboard with a yardstick. Compare the results.

Ask the students how to convert without actually measuring. (We multiply to convert larger units to smaller units and divide to find the reverse.)

7.10.01.04 **solve verbal problems related to skills learned in this module**

STRATEGY 1: Write the following problem on the chalkboard.

7.10.01

STRATEGY 1: cont'd

The distance from home plate to the left field fence is 437 feet. What is the measurement in yards?

Say, "What information is needed to solve this problem? (3 ft. = 1 yd.) Use this information to solve the problem." Ask a student to write the solution on the chalkboard.

e.g.,

$$3\overline{)\,437} = 145\frac{2}{3} \text{ yards}$$

Ask a student to give the answer to the nearest yard. Elicit 146 yards.

Repeat this procedure with other problems using measurements.

STRATEGY 2: Write this problem on the chalkboard.

Joe went to the doctor for a checkup. He needed to know his height and weight for the school nurse. His height was $5\frac{3}{4}$ feet and his weight was 156 pounds. When he gave this information to the nurse, he was asked to write his height in inches. What was Joe's height in inches?

Say that one of the techniques in solving problems is to use a table and list the information needed.

e.g.,

Feet	Inches
$\frac{1}{4}$	3
$\frac{1}{2}$	☐
$\frac{3}{4}$	☐
1	12
2	24
3	36

Ask the students to complete the table and use the information to solve the problem. Repeat this procedure with other problems.

P.O. 7.10.01.01

NAME: ______________________________ CLASS: __________ DATE: ________

Select the appropriate unit to complete each sentence. Choose from inch(es), feet, yard(s), mile(s).

1. The length of a baseball bat is 37 __________.

2. The length of a pencil is 7 ________.

3. An airplane can fly at an altitude of 6 __________.

4. My math book is 1 ________ thick.

5. The distance from Chicago to St. Louis is 276 ________.

6. The height of Mount Lafayette is 5249 _________.

7. The length of a picnic table is 2 _________.

8. I know a basketball player who is 6 _____ 9 _____ tall.

9. The diameter of a quarter is $\frac{11}{12}$ of an _____.

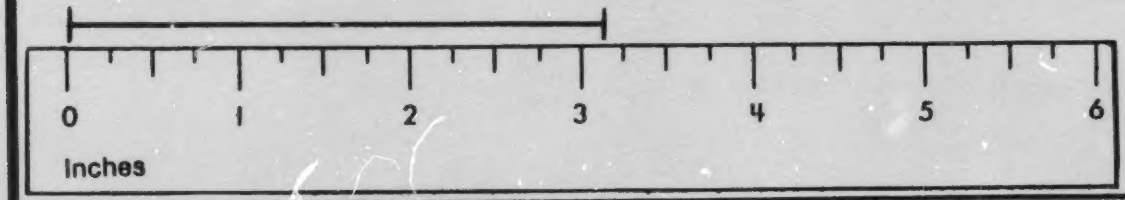
10. I am going to take part in the 100 _______ dash.

P.O. 7.10.01.02

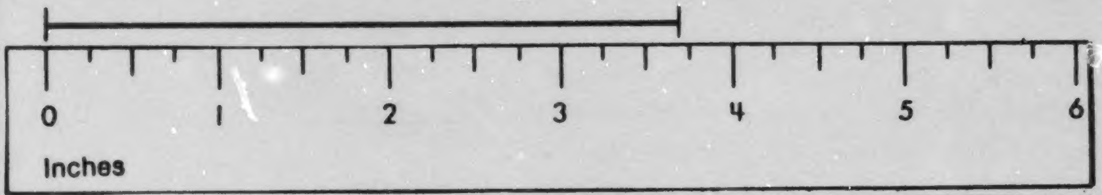
NAME:____CLASS:________DATE:_______

Determine the measures of the line segments.

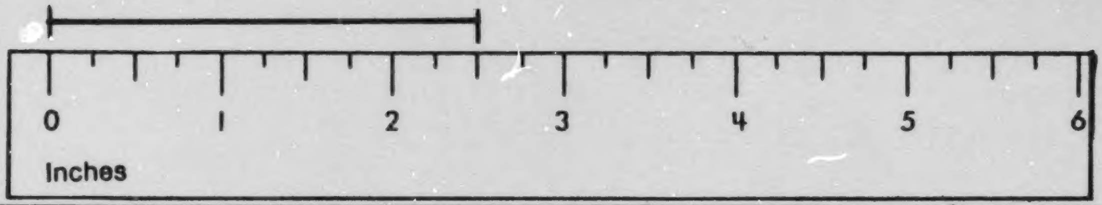
To the nearest inch __________



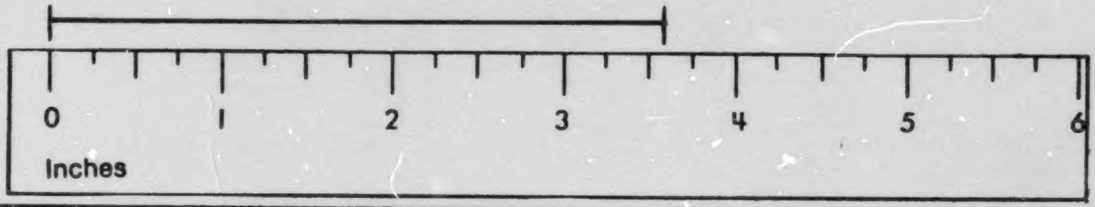
To the nearest ½ inch __________



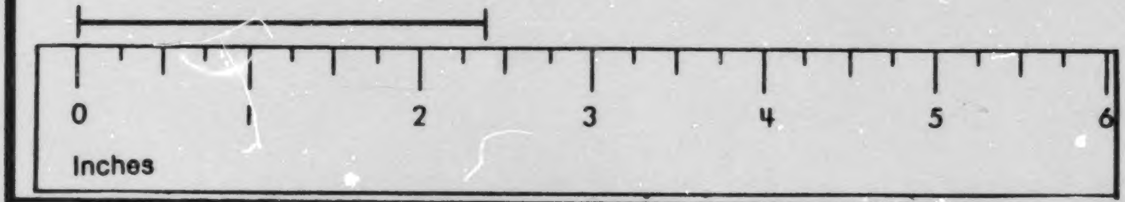
To the nearest ½ inch __________



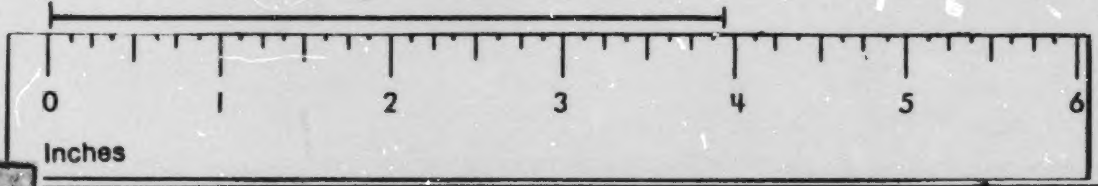
To the nearest ¼ inch __________



To the nearest ¼ inch __________



To the nearest ⅛ inch __________



P.O. 7.10.01.03

NAME:____ CLASS:_________ DATE:_______

Convert from one unit of length to the other.

1. 5 ft.= ☐ in.	2. 48 in.= ☐ ft.	3. 2 yd.= ☐ in.	4. 9 ft.= ☐ yd.
5. 15 yd.= ☐ ft.	6. 30 in.= ☐ ft.	7. 7 ft.= ☐ yd.	8. 7,040 ft.= ☐ mi.
9. 6 ft. 3 in.= ☐ in.	10. 4 yd. 2 ft. = ☐ ft.	11. 3 mi.= ☐ ft.	12. $\frac{2}{3}$ yd. = ☐ ft.
13. 3½ ft.= ☐ yd.	14. 24 in.= ☐ yd.	15. $1\frac{3}{4}$ ft.= ☐ in.	16. 1760 ft.= ☐ mi.

P.O. 7.10.01.04

NAME: ______________________ CLASS: __________ DATE: ________

Solve.

1. If 4 feet of lace will trim one dress, how many yards of lace are needed for 24 dresses?

2. The side of a square measures 14 inches. Express the perimeter to the nearest foot.

3. It takes $4\frac{1}{4}$ feet of trimming to trim one shelf. How many yards of trimming will be needed to trim 3 shelves?

4. A fence 15 yards long requires a post every 2 feet. How many posts will this fence need?

5. An ant fell to the bottom of a 10 foot deep hole. During the first hour the ant climbed up 7 inches and then slid back 4 inches. At that rate, how many hours did it take the ant to climb out of the hole? (***Be careful!!!)

TABLE OF SYMBOLS

DEFINITIONS	SYMBOL	DEFINITIONS	SYMBOL
plus; add; positive	+	line	⟷
minus; subtract; negative	-	ray	⟶
plus or minus	+ or -	perpendicular lines	⊥
times; multiply	· or X	parallel lines	∥
divide	÷	degree	°
divide	$\overline{)\quad}$	equals	=
square root	$\sqrt{\quad}$	is not equal to	≠
cube root	$\sqrt[3]{\quad}$	is less than	<
angle	∡	is greater than	>
between two numerals or letters at the end of a series or sequence; and so on	...	encloses a set when numbers are included; empty set when empty	{ }
		subset	⊂
triangle	△	intersection	∩
rectangle	▭	union; join	∪
parallelogram	▱	percent	%
circle	⊙	foot, minute	'
		inch, second	"
line segment (as $\overline{AB}$)	—		

METRIC TABLE

PREFIX	VALUE	LENGTH	CAPACITY	WEIGHT (MASS)
KILO	1000	kilometer* (km)	kiloliter (kl)	kilogram* (kg)
HECTO	100	hectometer (hm)	hectoliter (hl)	hectogram (hg)
DEKA	10	dekameter (dam)	dekaliter (dal)	dekagram (dag)
	1	meter* (m)	liter* (l)	gram* (g)
DECI	$\frac{1}{10}$ or .10	decimeter (dm)	deciliter (dl)	decigram (dg)
CENTI	$\frac{1}{100}$ or .01	centimeter* (cm)	centiliter (cl)	centigram (cg)
MILLI	$\frac{1}{1000}$ or .001	millimeter* (mm)	milliliter (ml)	milligram* (mg)

* most frequently used

BASIC CLASSROOM EQUIPMENT CHECKLIST

MATERIALS:

Abacus - Class size/individual. ____

Attribute blocks................ ____

Balance Beam.................... ____

Calculators..................... ____

Calendar - class size........... ____

Clock - large - class size...... ____

Clock - individual size......... ____

Clock - stamp................... ____

Coin stamps..................... ____

Compass......................... ____

Cubic counting blocks........... ____

Cuisenaire rods................. ____

Dominoes........................ ____

Felt cut-outs................... ____

Flannel board................... ____

Flash cards..................... ____

Fractional parts (assorted)..... ____

Geo boards...................... ____

Geometric patterns.............. ____

Hundred charts - class size/ind. ____

Laminating paper................ ____

Magnetic board.................. ____

Magnetic discs.................. ____

Measuring tools-English, Metric (rulers, scales, compasses, protractors, beakers, cups, spoons, thermometers)........ ____

Number lines - class/individual. ____

Numerals - tactile.............. ____

Plastic discs................... ____

Play money...................... ____

Powers of Ten................... ____

Protractor...................... ____

SEE calculators................. ____

Scale........................... ____

Screen (pull-down type)......... ____

Spinners........................ ____

Squared place value material.... ____

Standard playing cards.......... ____

Tangrams........................ ____

Tens frame...................... ____

Timers (1,2,3 minute)........... ____

Tokens (various sizes/colors)... ____

Trundle wheel................... ____

Unifix cubes.................... ____

Unifix 100 board................ ____

SUPPLIES:

Adding machine tapes (for classroom/ind. number lines)...... ____

Chalk........................... ____

Construction paper.............. ____

Elastic bands................... ____

BASIC CLASSROOM EQUIPMENT CHECKLIST

SUPPLIES: cont'd

Envelopes (all sizes).......... ____
Erasers, board/individual...... ____
Felt pens (all sizes).......... ____
Ditto, Rexograph paper......... ____
Glue........................... ____
Gram weights................... ____
Graph paper (1 cm, <, 1")...... ____
Grease pencils................. ____
Oak tag........................ ____
Paper clips.................... ____
Paste.......................... ____
Plastic discs.................. ____
Pointer........................ ____
Primary pencils................ ____
Rubber bands................... ____
Scissors - primary, left hand.. ____
Scotch tape.................... ____
Spinner........................ ____
String......................... ____
Tape........................... ____

COLLECTIBLES:

Beads and beans................ ____
Bottle caps.................... ____
Buttons........................ ____
Cardboard (all sizes).......... ____
Cereal boxes................... ____
Coffee cans.................... ____
Detergent boxes, shoe boxes,
meat trays, and plastic
containers..................... ____
Egg cartons.................... ____
Felt for desk mats............. ____
Ice cream sticks............... ____
Juice containers............... ____
Macaroni....................... ____
Magazines...................... ____
Menus.......................... ____
Metrically measured cartons.... ____
Milk containers................ ____
Newspapers..................... ____
Paper plates................... ____
Sand........................... ____
Spools......................... ____
String, yarn................... ____
Styrofoam...................... ____
Tongue depressors.............. ____
Wire hangers................... ____
Ziploc Bags.................... ____